UDC 621.8

Modeling dynamic control model of a two-link crane-manipulator

Volodymer Voliyanuk[1], *Dmitry Mishchuk*[2], *Maksym Parkhomenko*[3]

[1,2,3]Kyiv National University of Construction and Architecture
31, Povitroflotsky Ave., Kyiv, Ukraine, 03037
[1]volianiuk.vo@knuba.edu.ua, https://orcid.org/0000-0002-6852-9037,
[2]mischuk.do@knuba.edu.ua, https://orcid.org/0000-0002-8263-9400
[3]parkhomenko_ma@knuba.edu.ua

Received: 03.10.2022; Accepted: 28.11.2022
https://doi.org/10.32347/gbdmm.2022.99.0201

Abstract. The problem of controlling a two-link crane-manipulator depends on the level of complexity of the system. When using a two-link manipulator, two problems need to be solved. The first problem is that you need to define the control parameters in such a way that the given position of the manipulator changes according to the corresponding trajectory. The second problem is that in order to achieve the desired position of the manipulator, it is necessary to correctly parameterized the mathematical model of the control system. One of the problems that affects the accuracy of manipulator control is the correct construction of adequate dynamic control models.

In this article was propose to consider a direct method of developing a dynamic model of the crane-manipulator. It was propose to apply a mathematical model, which is built on the principles of theoretical mechanics by applying the Lagrange equations of the second kind.

To modeling a dynamic model of a two-link crane-manipulator, the kinetic and potential energies of the component systems of the manipulator were determined, and on the basis of the Lagrange equation of the second kind, dynamic equations of motion were obtained.

The dependences between the capturer coordinates and the generalized coordinates were also determined. These dependencies helped to establish the control equations that allow controlling the movement characteristics of the two-link manipulator.

Keywords: manipulator, Lagrange equation, kinetic energy, dynamic model, equation of motion.

INTRODUCTION

Recently, the use of manipulators and robots has become widespread in the construction industry [1]. Such systems significantly increase the speed of execution of technological construction processes, and significantly reduce material costs [2].

One main problems of effective application of robotic systems and cranes-manipulators is the use of adequate dynamic models of their control. In particular, one of the difficult tasks is the analytical control of the given trajectories of the movement of the working body [3, 4].

Developing a mathematical model of a two-link manipulator is one of the most important tasks in the development of robotics. With the aid of mathematical analysis, it was possible to create an adequate model of a two-link lifting system crane-manipulator, which will allow predicting its behavior in various conditions.

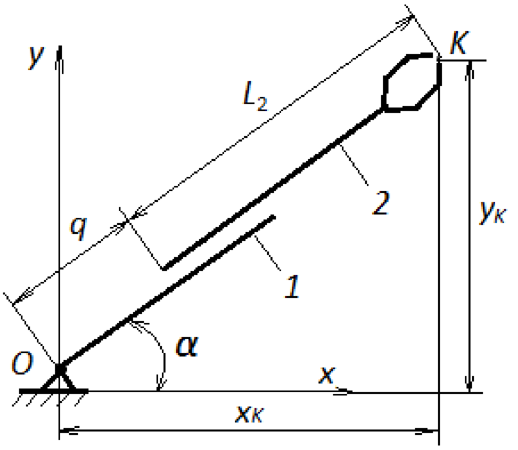
PURPOSE OF THE ARTICLE

Main purpose this article to develop the dynamic model of control of a two-link lifting boom system crane-manipulator, which would take into account the laws of control of the working body.

PRESENTING MAIN MATERIAL

Let's consider a listing boom system crane-manipulator consisting of two independently moving links *1*, *2* with actuators and a gripper K (see Fig. 1). The movement of the manipulator links is carried out by actuators *A* and *B*, which are not shown in the diagram under consideration.

a



b

Fig. 1. View of the jib lifting system of the two-link crane-manipulator (*a*) and its dynamic model (*b*)

In this dynamic scheme of the dynamic model of the manipulator, the following assumptions are made:

- m_1 – the mass of the first link, which is concentrated in the center of this link at a distance $1/2L_1$ from its turning point;
- m_2 – the mass of the second link and the moving object in the gripper, which is concentrated at a point *K*.

The rotation angle is taken as the generalized coordinates in this diagram of the two-link manipulator α of the first link relative to the horizontal surface and the coordinate *q* moving the second link relative to the first.

The kinetic energy of the considered kinematic system of the manipulator is determined as a function of generalized coordinates and generalized velocities.

The kinetic energy of link *1*, which rotates around point *O*, will be [5]:

$$T_1 = \frac{J_1 \omega_1^2}{2}, \qquad (1)$$

where:

J_1 - the moment of inertia of the rotary link relative to point O;

ω_1 - an angular velocity of the rotary link.

The kinetic energy of link *2*, which rotates around point *O* and simultaneously moves along the axis of link *1*, will be [5]:

$$T_2 = \frac{J_{2c}\omega_2^2}{2} + \frac{m_2 \upsilon_2^2}{2}, \qquad (2)$$

where:

J_{2c} - combined moment of inertia of the rotary link *2* relative to the point *O*;

ω_2 - an angular speed of rotation of the link *2*;

m_2 - weight of link 2 with load;

υ_2 - linear speed of movement of the link 2.

The speed of movement of link *2* is determined through projections of its speed [6]:

$$\upsilon_2 = \sqrt{\left(\frac{dx_K}{dt}\right)^2 + \left(\frac{dy_K}{dt}\right)^2}, \qquad (3)$$

where:

$x_K = (q + L_2)\cos\alpha$, $y_K = (q + L_2)\sin\alpha$.

The time derivatives of the velocity coordinates will be:

ISSN(online)2709-6149. Mining, constructional, road and melioration machines, 99, 2022, 15-19

$$\frac{dx_K}{dt} = \dot{x}_K = \dot{q}\cos\alpha - \dot{\alpha}(q+L_2)\sin\alpha\,, \quad (4)$$

$$\frac{dy_K}{dt} = \dot{y}_K = \dot{q}\sin\alpha + \dot{\alpha}(q+L_2)\cos\alpha\,, \quad (5)$$

Squares of velocities by projections

$$\begin{aligned}\dot{x}_K^2 &= \dot{q}^2\cos^2\alpha + \dot{\alpha}^2(q+L_2)^2\sin^2\alpha - \\ &-2\dot{q}\dot{\alpha}\cos\alpha(q+L_2)\sin\alpha;\end{aligned} \quad (6)$$

$$\begin{aligned}\dot{y}_K^2 &= \dot{q}^2\sin^2\alpha + \dot{\alpha}^2(q+L_2)^2\cos^2\alpha + \\ &+2\dot{q}\dot{\alpha}\sin\alpha(q+L_2)\cos\alpha.\end{aligned} \quad (7)$$

Taking into account the dependence (6), (7) and (3), the square speed of link *2* can be determined through generalized speeds

$$\upsilon_2^2 = \dot{q}^2 + (q+L_2)^2\dot{\alpha}^2\ . \quad (8)$$

So, as the angular velocity of link *2* coincides with the angular velocity of link *1*, we denote:

$$\omega_1 = \dot{\alpha} \quad \omega_2 = \dot{\alpha}\,. \quad (9)$$

The kinetic energy of the boom system of the considered two-link lifting boom system crane-manipulator will be:

$$\begin{aligned}T &= T_1 + T_2 = \frac{J_1\dot{\alpha}^2}{2} + \frac{J_{2c}\dot{\alpha}^2}{2} + \\ &+\frac{m_2(\dot{q}^2 + (q+L_2)^2\dot{\alpha}^2)}{2}.\end{aligned} \quad (10)$$

According to Steiner's theorem, we will accept the following assumption for link *2* [7]:

$$J_{2c} = J_2 + m_2\rho^2\,, \quad (11)$$

then for $\rho = (q+L_2)$, was obtain the following expression of the kinetic energy of the system under consideration:

$$T = \frac{J_\Sigma\dot{\alpha}^2}{2} + \frac{m_2\dot{q}^2}{2} + m_2(q+L_2)^2\dot{\alpha}^2, \quad (12)$$

where:

$J_\Sigma = (\frac{1}{3}m_1L_1^2 + \frac{1}{12}m_2L_2^2)$ - the moment of inertia of the boom system is presented;

L_1, L_2, m_1, m_2 - length and mass of the corresponding components of the manipulator's mechanical system.

In accordance with the selected generalized coordinates of a mechanical system with two degrees of mobility, we have the following system of second-order Lagrange equations:

$$\begin{cases}\dfrac{d}{dt}\left(\dfrac{\partial T}{\partial\dot{\alpha}}\right) - \dfrac{\partial T}{\partial\alpha} = Q_\alpha; \\ \dfrac{d}{dt}\left(\dfrac{\partial T}{\partial\dot{q}}\right) - \dfrac{\partial T}{\partial q} = Q_q,\end{cases} \quad (13)$$

where:

Q_α, Q_q - generalized forces.

The components of the above equation was define:

$$\frac{\partial T}{\partial\alpha} = 0\,; \quad (14)$$

$$\frac{\partial T}{\partial\dot{\alpha}} = J_\Sigma\dot{\alpha}^2 + 2m_2\dot{\alpha}(q+L_2)^2\,; \quad (15)$$

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\alpha}}\right) &= J_\Sigma\ddot{\alpha} + 2m_2\ddot{\alpha}(q+L_2)^2 + \\ &+4m_2\dot{\alpha}(q+L_2)\dot{q};\end{aligned} \quad (16)$$

$$\frac{\partial T}{\partial q} = 2m_2\dot{\alpha}^2(q+L_2)\,; \quad (17)$$

$$\frac{\partial T}{\partial\dot{q}} = m_2\dot{q}\,; \quad (18)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\alpha}}\right) = m_2\ddot{q}; \quad (19)$$

Substitute the resulting dependencies (14) – (19) into equation (13) and obtain the dynamic equation of motion of the two-link manipulator

$$\begin{cases}J_\Sigma\ddot{\alpha} + 2m_2\ddot{\alpha}(q+L_2)^2 + 4m_2\dot{\alpha}\dot{q}(q+L_2) = \\ = M - \dfrac{m_1gL_1}{2}\cos\alpha - m_2g(q+L_2)\cos\alpha; \\ m_2\ddot{q} - 2m_2\dot{\alpha}^2(q+L_2) = P - m_2g\sin\alpha,\end{cases} \quad (20)$$

where:
M, *P* – the driving moment of the movement of link *1* and the driving force of the movement of link *2*.

The dependence of the generalized coordinates on the coordinates of the position of the cargo was found:

$$\alpha = \arctan(\frac{y_k}{x_k}); \quad (21)$$

$$q = \frac{x}{\cos(\arctan(\frac{y_k}{x_k}))} - L_2. \quad (22)$$

The first time derivative of expressions (21) and (22) will have the following form

$$\dot{\alpha} = \frac{-y_k \dot{x}_k + x_k \dot{y}_k}{x_k^2 + y_k^2}; \quad (23)$$

$$\dot{q} = \frac{x_k \dot{x}_k + y_k \dot{y}_k}{x_k \sqrt{1 + \frac{y_k^2}{x_k^2}}}. \quad (24)$$

Derivatives of the second order will have the form, respectively:

$$\ddot{\alpha} = \frac{1}{\left(x_k^2 + y_k^2\right)^2}(y_k^2(2\dot{x}_k \dot{y}_k - y_k \ddot{x}_k) - x_k^2(2\dot{x}_k \dot{y}_k + y_k \ddot{x}_k) + \dot{x}_k^3 \ddot{y}_k + x_k y_k (2\dot{x}_k^2 - 2\dot{y}_k^2 + y_k \ddot{y}_k)) \quad (25)$$

$$\ddot{q} = \frac{1}{x_k\left(x_k^2 + y_k^2\right)\sqrt{1 + \frac{y_k^2}{x_k^2}}}(y_k^2(\dot{x}_k^2 + x_k \ddot{x}_k) + x_k^2(\dot{y}_k^2 + x_k \ddot{x}_k) + y_k^3 \ddot{y}_k + x_k y_k(-2\dot{x}_k \dot{y}_k + x_k \ddot{y}_k)). \quad (26)$$

CONCLUSIONS

Analyzing equation (20), we note that if the derivatives of the generalized coordinates of the manipulator are replaced by expressions (25) and (26), then it will be possible to obtain the dependences of the driving moment and the driving force for this mathematical model of the manipulator.

REFERENCES

1. **Loveikin V., Mishchuk D.** (2022). Analysis of the refined mathematical model of the manipulator boom system with an elastic base. Girnychi, budivelni, dorozhni ta melioratyvni mashyny [Mining, construction, road and melioration machines], Nr99, 5-14.
2. **Mischuk D. Volianiuk V., Gorbatyuk Eu.** (2019). Determination of geometric parameters of the manipulator according to the characteristics of the working environment. Girnichi, budivelni, dorozhni ta meliorativni mashini [Mining, construction, road and reclamation machines], Nr.94, 25-34. (*in Ukrainian*). https://doi.org/10.32347/gbdmm2019.94.0201
3. **Loveikin V. S., Mishchuk D. O.** (2012). Mathematical modeling of changes in the departure of cargo by a manipulator with a hydraulic drive. Girnychi, budivelni, dorozhni ta melioratyvni mashyny [Mining, construction, road and melioration machines], Nr79, 9-15.
4. **Volianiuk V., Mischuk D., Gorbatyuk Eu.** (2022). Determination of dynamic loads in the crane lifting mechanism. Girnichi, budivelni, dorozhni ta meliorativni mashini [Mining, construction, road and reclamation machines], Nr.100, 12-22. (*in Ukrainian*).
5. **Volianiuk V., Mischuk D., Gorbatyuk Eu.** (2020). Determination of inertial loads of the rotary boom of a self-propelled crane. Girnichi, budivelni, dorozhni ta meliorativni mashini [Mining, construction, road and reclamation machines], Nr.96, 13-21. (*in Ukrainian*). https://doi.org/10.32347/gbdmm2020.96.0201
6. **Loveykin V. S., Mischuk D. O.** (2013). Optimization of the modes of changing the departure of the manipulator with a hydraulic drive. Monograph, Kyiv, CP Komprint, 206. https://doi.org/10.26884/damu.m13opzvmg
7. **Mishchuk D. O.** (2017). Study of the dynamics of the boom manipulator mounted on an elastic sup-port. Girnychi, budivelni, dorozhni ta melioratyvni mashyny [Mining, construction, road and melioration machines], Nr90, 11-18.

Розробка динамічної моделі управління дволанковим краном-маніпулятором

Володимир Волянюк[1], *Дмитро Міщук*[2], *Максим Пархоменко*[3]

[1,2,3]*Київський національний університет будівництва і архітектури*

ISSN(online)2709-6149. Mining, constructional, road and melioration machines, 99, 2022, 15-19

Анотація. Проблема керування дволанковим маніпулятором залежить від рівня складності системи. При використанні дволанкового маніпулятора потрібно вирішити дві проблеми. Перша проблема полягає в тому, що потрібно визначити параметри керування таким чином, щоб задана позиція маніпулятора змінювалась за відповідною траєкторією. Друга проблема полягає в тому, що для досягнення потрібної позиції маніпулятора потрібно правильно параметризувати математичну модель системи управління. Однією з проблем, яка впливає на точність управління маніпуляторами полягає в правильній побудові адекватних динамічних моделей керування.

В даному дослідженні запропоновано розглянути прямий метод розробки динамічної моделі маніпулятора. Пропонується застосовувати математичну модель, яка побудована на принципах теоретичної механіки із застосування рівнянь Лагранжа другого роду.

Для створення динамічної моделі дволанкового маніпулятора було визначено кінетичну та потенціальні енергії складових систем маніпулятора, а на основі рівняння Лагранжа другого роду отримано динамічні рівняння руху.

Також було визначено залежності між координатами захоплювача та узагальненими координатами. Ці залежності допомогли встановити рівняння управління, які дозволяють здійснити керування за характеристиками руху захоплювача дволанкового маніпулятора.

Ключові слова: маніпулятор, рівняння Лагранжа, кінетичні енергія, динамічна модель, рівняння руху.