

Mathematical model of heat distribution in an abrasive wheel

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Abstract. The cost of abrasive cutting is mainly determined by the wear resistance of the abrasive wheel, which consists of abrasive grain, filler, phenolic binder and glass mesh. Due to the fact that during the cutting process, as a result of the summation of heat pulses from the cutting grains located on the cutting edge of the wheel, a significant amount of heat is released, high temperature values are reached in the cutting zone. Meanwhile, it is well known that the phenolic binder has low heat resistance, it breaks down at a temperature of 520-570 °K, therefore the nature of the thermal processes occurring during abrasive cutting determines the temperature in the circle and, accordingly, the speed of its wear. Ideally, of course, the speed of thermal destruction of the bond should correlate with the speed of mechanical destruction of abrasive grains, so that cutting is carried out only with sharp, unfired grains, while only blunt grains should be removed from the cutting edge. Since the rate of abrasion of abrasive grains is different for different processed materials, the characteristics of binders must also depend on the type of processed material, that is, it is necessary to create abrasive wheels for cutting different materials. In practice, abrasive wheels are produced without special consideration of the features of the cut material, which is largely explained by the lack of clarity of the nature of thermal processes in abrasive reinforced wheels and technological difficulties associated with changing the thermophysical properties of the wheels.

Keywords: abrasive reinforced wheels, cutting, high-abrasive materials, temperature, connection, lateral surfaces

INTRODUCTION

Let's dwell in more detail on the thermal destruction of the bond. The phenol bond (Bakelite) is polycot-densified in the process of heat treatment. When they say that when heated to a certain temperature, it loses its strength, they rely on the experimentally established fact - if you heat polycondensation Bakelite to this temperature, it burns. This heating process is quite slow, so you can confidently use such an equilibrium characteristic as temperature.

When cutting with an abrasive wheel, the picture changes. In practice, the lengths of the arcs of contact are small, and the circles rotate at a high speed. Therefore, the residence time of a separate section of the cutting edge in the area where the energy is released is very short. Outside the cutting zone, due to heat transfer to the surrounding air, the wheel cools. Such shock alternation of heat loads and cooling leads to the fact that the system is strongly unbalanced, in which the very concept of temperature should be used with caution.

Therefore, a characteristic similar to the experimentally determined temperature of destruction of a Bakelite bond will not be an instantaneous temperature, which, moreover, cannot be measured due to the inertia of measuring devices, but some averaged temperature. It is advisable to carry out such averaging over the period of rotation, that is, to determine the temperature of a given point in the circle, averaged over the period of rotation and therefore independent of the rotation of the circle [1-4].

Studying in this way a certain temperature and the influence on it of such technologically important variables as the speed of rotation and feed of the wheel, its dimensions, the structure of the side surface, the length of the arc of contact and the thermophysical characteristics of the material of the wheel will allow to determine the rate of thermal wear of the wheel.

Assuming that the depth of heat penetration into the matrix of an abrasive reinforced wheel is significantly less than its radius, we limit ourselves to considering thermal processes in a narrow region adjacent to the cutting edge of the wheel. At the same time, taking into account the ratio of values characterizing the length of the arc of contact \tilde{l}_k , the height of the circle \tilde{H} and its radius R_0 , we neglect the curvature of the cutting zone.

Thus, the theoretical study of thermal processes can be reduced to the solution of the problem of heat distribution in a rectangular semi-infinite plate with a thickness equal to the height of a circle, along one side of which a flat source of power \tilde{q} , length \tilde{l}_k and width \tilde{H} moves with a linear speed of rotation of the circle $V_p = \omega R_0$. Let's introduce a rectangular coordinate system, the origin of which coincides with the source (Fig. 1).

Heat conduction equation in the case of a moving source

$$\frac{\partial \tilde{T}}{\partial t} = a \Delta \tilde{T} + V_p \frac{\partial \tilde{T}}{\partial \tilde{Y}}, \quad (1)$$

where:

\tilde{T} - is the temperature of the polymer matrix, K;

$a = \frac{\lambda}{c\rho_{kp}}$ - coefficient of thermal conductivity

of the circle, m^2/c ;

λ - specific heat capacity of the circle material, $j/kg \cdot K$;

Δ - the Laplace operator;

$\Delta = \frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{y}^2} + \frac{\partial^2}{\partial \tilde{z}^2}$; \tilde{x} \tilde{y} \tilde{z} - coordinates,

m;

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t - time, s.

Having established dimensionless complexes, we have:

$$\frac{\partial T}{\partial \theta} = \frac{1}{Pe} \Delta T + \frac{\partial T}{\partial Y}, \quad (2)$$

where:

$$\theta = \frac{tV_p}{R_0}, \quad T = \frac{\tilde{T}}{\tilde{T}_k}, \quad x = \frac{\tilde{x}}{R_0}, \quad (3)$$

$$y = \frac{\tilde{y}}{R_0}, \quad z = \frac{\tilde{z}}{R_0},$$

\tilde{T}_k - temperature in the contact zone of the circle with the processed object, K;

$Pe = \frac{V_p R_0}{a}$ - the Péclet criterion, characterizing the ratio of the speed of movement of the heat source to the speed of heat transfer.

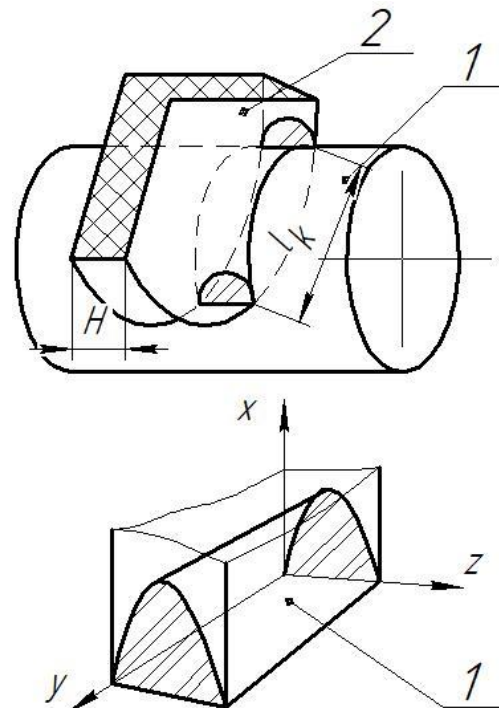


Fig. 1. Heat source (1) moving at speed along the cutting edge of the circle (2)

Boundary conditions take into account the release of heat in the contact zone

$$-\frac{\partial T}{\partial x}\Big|_{\substack{x=0 \\ y \in [0, l_k]}} = q \quad (4)$$

and heat transfer to the environment outside this contact

$$\frac{\partial T}{\partial x}\Big|_{\substack{x=0 \\ y \notin [0, l_k]}} - BiT\Big|_{\substack{x=0 \\ y \notin [0, l_k]}} = 0, \quad (5)$$

$$\frac{\partial T}{\partial z}\Big|_{z=0} - BiT\Big|_{z=0} = 0, \quad (6)$$

$$\frac{\partial T}{\partial z}\Big|_{z=0} - BiT\Big|_{z=0} = 0, \quad (7)$$

where:

$q = \frac{\tilde{q}R_0}{\lambda\tilde{T}_k}$ – heat flux density, $[\tilde{q}] = \text{W/m}^2$;

$Bi = \frac{\lambda_0 R_0}{\lambda}$ – the Bio criterion, characterizing the ratio of the rate of heat transfer to the rate of heat supply to the surface of the circle;

$H = \frac{\tilde{H}}{R_0}$; $l_k = \frac{\tilde{l}_k}{R_0}$ are dimensionless values of

the height of the circle and the length of the arc of contact, respectively.

In (5) it is taken into account that the source is flat $x' = 0$ and the temperature along the width of the source is constant and equal T_k . β_n is determined from the solution of the transcendental equation.

$$\text{ctg}(\beta_n H) = \frac{\beta_n}{Bi} \quad (8)$$

After calculating the integrals, we determine the temperature distribution law in the circle matrix:

$$\begin{aligned} \tilde{T}_i &= \frac{P_l^{5/2} \tilde{q} \tilde{l}_k}{\sqrt{\pi} Bi^2 c \rho_{kp} \tilde{H} V_p} \cdot \\ &\cdot \sum_{n=1}^{\infty} \frac{\sin(\beta_n H) \cos(\beta_n z)}{\beta_n} \cdot \\ &\cdot \frac{\exp\left(-\beta_n \frac{|y|}{P_l} - \frac{x^2 P_l}{|y|}\right)}{|y|^{3/2}} \left[Bix + \sqrt{|y|} \right] \end{aligned} \quad (9)$$

The resulting formula describes the temperature distribution in the abrasive circle for one revolution. At the same time, the abrasive grain makes a significantly greater number of revolutions before painting from the polymer matrix of the circle, that is, it is exposed to the action of the corresponding number of heat pulses. In the accepted model N_0 , the revolutions of the abrasive wheel correspond N_0 to the same heat pulses, which are separated from each other by a distance $\tilde{y} = 2\pi R$.

The maximum values of the temperature determining the wear resistance of the circle are reached in its middle plane, so we take $z = \frac{H}{2}$. As a result, we obtain a dependence that allows us to calculate the temperature of the polymer matrix of the circle at any distance from its cutting edge:

$$\tilde{T}(x) = \sum_{i=1}^{N_0} \tilde{T}_i, \quad (10)$$

where:

$$\begin{aligned} \tilde{T}_i &= \frac{P_l^{5/2} \tilde{q} \tilde{l}_k}{2\sqrt{2}\pi^2 Bi^2 c \rho_{kp} \tilde{H} V_p} \cdot \\ &\cdot \sum_{n=1}^{\infty} \frac{\sin(\beta_n H) \cos\left(\beta_n \frac{H}{2}\right)}{\beta_n} \cdot \\ &\cdot \frac{\exp\left(-\beta_n \frac{2\pi i}{P_l} - \frac{x^2 P_l}{2\pi i}\right)}{|y|^{3/2}} \left[Bix + (2\pi i)^{1/2} \right] \end{aligned} \quad (11)$$

Let's determine the number of revolutions of the circle at which its radius will decrease by an amount equal to the size of the abrasive grain δ . We assume that the circle moves with the feed rate V_n and goes deeper h to the cutting depth of the object.

According to (5), the abrasive wheel in time dt processes the object with an area of, and wears out by the amount $dF_g = hV_n dt$

$$dF_{kp} = 2\pi R dR = \frac{dF_g}{S_p} \quad (12)$$

From here:

$$hV_n dt = 2\pi S_p R dR \quad (13)$$

$$\int_0^t hV_n dt = 2\pi \int_{R_0-\delta}^{R_0} S_p R dR \quad (14)$$

The time for which the abrasive wheel wears from radius R_0 to radius $R_0 - \delta$:

$$t = \frac{2\pi}{hV_n} \int_{R_0-\delta}^{R_0} S_p R dR \quad (15)$$

The number of revolutions that the circle makes in time t is equal to an integral part of the value $t\omega = \frac{tV_p}{R_0}$

$$N_0 = \left[\frac{2\pi V_p}{hV_n R_0} \int_{R_0-\delta}^{R_0} S_p R dR \right] \quad (16)$$

Considering, $\delta \ll R_0$ and accepting $S_p = S_0 = \text{const}$ we have

$$\begin{aligned} N_0 &= \left[\frac{\pi V_p}{hV_n} S_0 (R_0^2 - (R_0 - \delta)^2) \right] \approx \\ &\approx [2\pi S_0 V_p \delta] \end{aligned} \quad (17)$$

Thus, according to (17), it is possible to determine the number of cuts that cut the abrasive grain until it breaks off from the matrix of the circle, and with the help of (12) and (16) to set the temperature at any point of the circle at a fixed feed rate.

The depth of heat penetration into the polymer matrix was determined experimentally to determine the dependencies between the modes of operation of the wheel, the power of the heat source \tilde{q} and the values of the heat transfer coefficient α_0 . The temperature measurement scheme is shown in Fig. 2 [1].

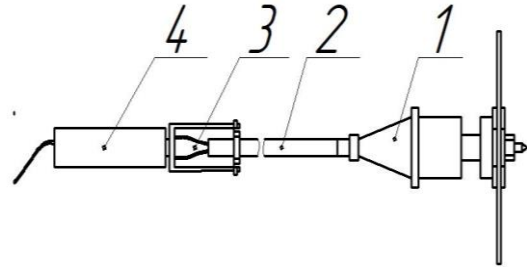


Fig. 2. Scheme of measurement of bond temperature of an abrasive reinforced wheel: 1 – head of a horizontal milling machine; 2 – rod; 3 – sleeve; 4 – mercury current collector.

When conducting experimental studies, abrasive reinforced circles made of normal electrocorundum 13A, 14A with a grain size of 320-800 microns were used. Such circles are used to perform cutting and cleaning operations in assembly and repair works with working speeds of 40-80 m/s and feeds of $2 \cdot 10^3 \dots 7 \cdot 10^3$. In this regard, dependencies (18), (19) and, respectively, (12 and 16) are valid only for the given characteristics of the abrasive wheel and operating modes.

To compare the theoretical and experimental data, the thermophysical characteristics of the abrasive wheel and the temperature in the contact zone were established.

As a result of experimental data processing, the following dependencies were obtained:

$$\begin{aligned} \tilde{q} &= 1,161 \cdot 10^9 - 1,375 \cdot 10^7 V_p + \\ &+ 0,4598 \cdot 10^{11} V_n \end{aligned} \quad (18)$$

$$\alpha_0 = 0,2531 \cdot 10^{-2} + 0,4971V_p, \quad (19)$$

$$\begin{aligned} \tilde{T}_\kappa = & 0,6935 \cdot 10^3 + 5,332V_p + \\ & + 5,997 \cdot 10^3V_n + 0,4028 \cdot 10^5 \tilde{H} \end{aligned} \quad (20)$$

The difference between the values of temperatures obtained theoretically and experimentally does not exceed 15% [6].

On the basis of the conducted studies, the degree of influence of the operating modes and design parameters of the abrasive reinforced wheel on its operational indicators was determined, as well as possible ways of increasing them.

The analysis of the obtained dependencies shows that the temperature of the polymer matrix of the abrasive wheel is related to the speed of its rotation through several parameters. As the speed increases in the contact zone, the number of thermal pulses increases, which negatively affects the wear resistance of the wheel. But at the same time, the heat transfer from its side surfaces increases, determined by the heat transfer coefficient [6]

$$N_u = 0,0296\sqrt[3]{P_r}(\text{Re})^{0,8}, \quad (21)$$

where: $P_r = 0,7$ is the Prandtl number for air; Re – Reynolds number;

$$\text{Re} = \frac{V_p R_0}{\nu_g}, \quad (22)$$

where: $\nu_g = 1,4 \cdot 10^{-5} \text{ m}^2/\text{s}$ is the kinematic viscosity of air.

As can be seen from (21), with growth V_p , the efficiency of heat transfer to the environment increases, which contributes to increasing the wear resistance of the wheel, as well as reducing the time it stays in the high temperature zone. Thus, increasing the working speed of the abrasive reinforced wheel is expedient.

The least favorable conditions for heat transfer are created in circles with a smooth side surface. The presence of irregularities contributes to more intense cooling of the side

surface of the circle due to the disruption of the laminar sublayer and the development of turbulence in the boundary layer. In the case of rough side surfaces, turbulent eddies appear around the protruding abrasive grains and heat transfer is carried out by convection, which, according to data, allows to increase the heat transfer coefficient up to two times compared to circles with smooth surfaces. The wear resistance of the abrasive wheel can also be increased as a result of intensifying heat dissipation by increasing the thermal conductivity of the polymer matrix or its forced cooling.

With an increase in the feed rate and an increase in the deformation of the chips, a greater amount of heat is released, part of which penetrates into the processed object. Based on this, it is necessary to determine the optimal amount of feed, which ensures the maximum wear resistance of the abrasive tool and the necessary quality of the cutting surface.

It was established that with the increase in the length of the arc of contact of the circle with the processed object, the heat generation increases. It should be taken into account that the circle is heated in the contact zone, and cooled outside it due to heat transfer to the environment. In this regard, the wear of the abrasive tool decreases when the ratio between the length of the arc of contact and the length of the cutting edge of the circle outside the contact is reduced.

It should also be taken into account that there are two types of cutting using abrasive reinforced wheels - dry and wet. Damp is used primarily to limit the amount of dust. When cutting with water supplied to the cutting zone for the purpose of dust removal, the temperature of the matrix decreases, fundamentally changing the wheel wear mechanism. The results of comparative temperature tests during cutting without water supply and with dust removal of the cutting process are shown in Fig. 3.

It can be seen from the graph (Fig. 3) that when working without water supply, the temperature rises by 40-400%, depending on the supply. During the supply of water to the cutting zone, the temperature does not significantly change from the supply force, while when

working "dry" the temperature and supply increase significantly.

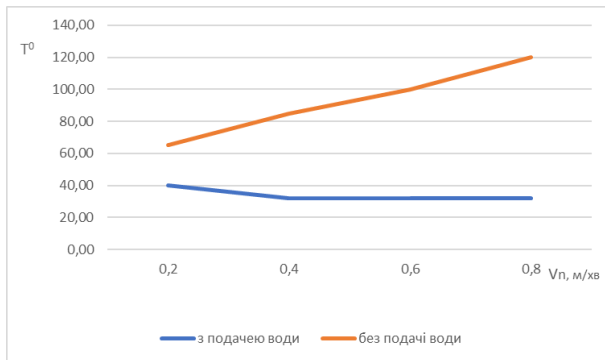


Fig. 3. Dependence of the bond temperature of the circle on the feed rate at a constant cutting speed $V_p = 66$ m/c

During the research, it was determined that the temperature value at the "grain-bond" boundary, which ensures thermomechanical wear of the wheel, should be 400-500K. Based on this, it was proposed to use a polymer matrix modifier. For this, substances whose heat resistance did not exceed the specified temperatures were selected: polyvinyl acetate (PVA) – 304K, polyvinyl alcohol (PVA) 413K, polyvinyl formal (PVC) – 365K, polyvinyl butyral (PVB) – 325K [5].

At the first stage, they were introduced directly into the composition of the abrasive mass after moistening the grains with liquid bakelite. The circles made using this technology showed unstable operation during the tests, as they do not self-sharpen during the cutting process.

CONCLUSIONS

In this regard, a new method of manufacturing abrasive reinforced wheels was developed, which consists in the fact that the abrasive grains are pre-coated with a 19-20% aqueous solution of polyvinyl acetate in the ratio of 0.8-5.4 g of solution per 1 kg. grain, which was evenly distributed over the grain surface. When cutting with circles made according to our technology, at the same time as the mechanical breaking of abrasive grains from the polymer matrix, its thermal destruc-

tion also occurs, i.e. the mechanism of wear is thermomechanical.

Circles modified with polyvinyl acetate are self-sharpening and allow cutting rocks and refractories with a strength of up to 60 MPa.

In the course of the work performed, a patent of Ukraine was obtained for the utility model Pat. 73906 Ukraine, IPC B24D 3/00 / Method of manufacturing an abrasive tool / Abrashkevich Y.D., Pelevin L.E., Polishchuk A.G., - No. u2012 03848 application. 29.03.2012; published 10.10.2012, Bull. No. 19/2012. On the basis of the received patent and the developed recipe of the wheel, samples of Abrasive reinforced wheels for cutting highly abrasive refractory materials with a strength of up to 60 MPa were produced. The circles have passed experimental and field tests.

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Математична модель розподілу теплоти в абразивному крузі

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Анотація. Собівартість абразивної різки в основному визначається зносостійкістю абразивного круга, що складається з абразивного зерна, наповнювача, фенольного сполучного і склосітки. У зв'язку з тим, що в процесі різання в результаті підсумовування теплових імпульсів від ріжучих зерен, які перебувають на ріжучій кромці круга, виділяють значну кількість тепла, в зоні різання досягаються великі значення температури. Тим часом добре відомо, що фенольна сполучна володіє низькою теплостійкістю, вона руйнується при температурі

520-570 °К, тому характер теплових процесів, що протікають при абразивному різанні, визначає і температуру в крузі і, відповідно, швидкість його зносу. В ідеалі, звичайно, швидкість теплового руйнування зв'язки повинна корелювати зі швидкістю механічного руйнування абразивних зерен з тим, щоб різання здійснювалося лише гострими, неспрацьовану зернами, при цьому усуватися з ріжучої кромки повинні лише тупі зерна. Оскільки швидкість стирання абразивних зерен різна для різних оброблюваних матеріалів, то й характеристики зв'язуючих повинні бути в залежності від виду оброблюваного матеріалу, тобто необхідно створювати абразивні круги для різки різних матеріалів. На практиці ж випускаються абразивні круги без особливого урахування особливостей розрізаного матеріалу, що в значній мірі пояснюється неясністю характеру теплових процесів в абразивних армованих кругах і технологічними складнощами, пов'язаними зі зміною теплофізичних властивостей кругів..

Ключові слова: абразивні армовані круги, різання, високоабразивні матеріали, температура, зв'язка, бічні поверхні.