UDC 621.87

Optimal mode movement of the robot manipulator on an elastic base according to the criterion of the mean square value of the acceleration of the drive torque

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Received: 23.04.2023; Accepted: 27.06.2023
https://doi.org/10.32347/gbdmm.2023.101.0201

Abstract. Previous studies of the optimization of the manipulator motion mode on an elastic base based on the criteria of the rms value of the drive torque and the rate of change of the rms value of the drive torque established that the use of optimal manipulator motion modes, which are created on the basis of higher derivatives of the characteristics of the change of the drive torque, allow to obtain more smooth movements of the boom system of the manipulator, thereby reducing its oscillations on the elastic support. At the same time, the obtained optimal modes of movement didn't completely eliminate the boom system oscillations during the change of departure. By observing the oscillations of the elastic link of the manipulator, the quality of the obtained optimal modes of movement was evaluate and the presence of residual oscillations was shown.

This work considers the possibility of optimizing the manipulator's motion mode based on the criterion of the mean square value of the acceleration of the drive torque change. For this purpose, the second derivative of the driving torque function of the manipulator boom system drive was determined first. Such a criterion for optimizing the motion mode is an integral functional, the minimization of which is carry out by the methods of variational calculus. The regularity of the change of the driving torque it was find from the dynamic equation of motion.

The search for the optimal motion mode of the manipulator realized through the solution of the Poisson boundary nonlinear differential equation of the fourth order.

The results of the conducted research made it possible to clarify much better the nature of the vibrations of the manipulator boom system and, as a result, to develop a drive control system that allows the realization of the obtained optimal mode of movement.

Keywords: manipulator, elastic base, optimization criterion, rms value of moment acceleration, minimization of oscillations.

INTRODUCTION

Manipulator dynamics modeling in many cases is carried out with the assumption that all links of its mechanical system and support mechanism are absolutely solid bodies, and the support surfaces are horizontal [1-5]. For mobile machines that work on different types of support surfaces, the mechanical properties of which are not known in advance, this assumption is false. It is important for manipulators of mobile robots to be able to minimize the oscillations of their support system [6], as this will affect the safety of work.

The oscillations of the boom system of the manipulator strongly depend on the magnitude and nature of the change in the driving torque of the drive mechanism. The regularity of the change in driving torque over time [7] has a decisive influence on the dynamics of the manipulator movement process.

ANALYSIS OF PUBLICATIONS

The analysis of known works it was shows the relevance of the existing problem of manipulator dynamics with elastic characteristics of links and support surfaces [8, 9].

The dynamics of the boom system of the manipulator on an elastic base was considered by a number of authors in works [6, 9-12], but a qualitative assessment of the influence of the drive system on the dynamics of the elastic support mechanism was not investigated. In work [12] it was research the problem of overloading and overheating of the power drive of the manipulator, which arises as a result of the action of dynamic loads, which are formed in the process of starting and braking due to the action of vibrations of the elastic parts of the boom and accumulate during work, is investigated. The author of the work [13] proposes an energy balancing method elastic deformations of the metal structure of the manipulator boom, which occur during fluctuations of unbalanced masses. The paper [14] considered the stability conditions of the mechanical manipulator system using the Lyapunov function. As a criterion for optimization in the study of boom systems of manipulators with flexible links, it is also possible to use the function of energy accumulation by its elastic element during the spatial movement of the load [10-12].

In work [15] was consider the method of synthesis of the optimal mode of motion of a manipulator on an elastic base according to kinematic characteristics, and in work [16] - according to the function of the driving moment. From the analysis of the conclusions of which it was clear that in order to optimize the modes of motion of the manipulator with the minimization of its oscillations elastic support mechanism. It is necessary to apply a criterion that will be built on the derivatives of the driving torque of the drive over time.

In this research, it is propose to consider the optimization problem according to the criterion of the minimum root mean square value of the change in the acceleration of the driving torque of the manipulator drive with an elastic base part and two generalized coordinates.

PURPOSE OF THE WORK

The main goal of the study is to minimize the oscillations of the manipulator links on an elastic base by optimizing the drive mechanism movement mode in the process of changing the load departure.

PRESENTING MAIN MATERIAL

A simplified dynamic model of the boom system of a two-link manipulator with an elastic support part is known (Fig. 1) [17].

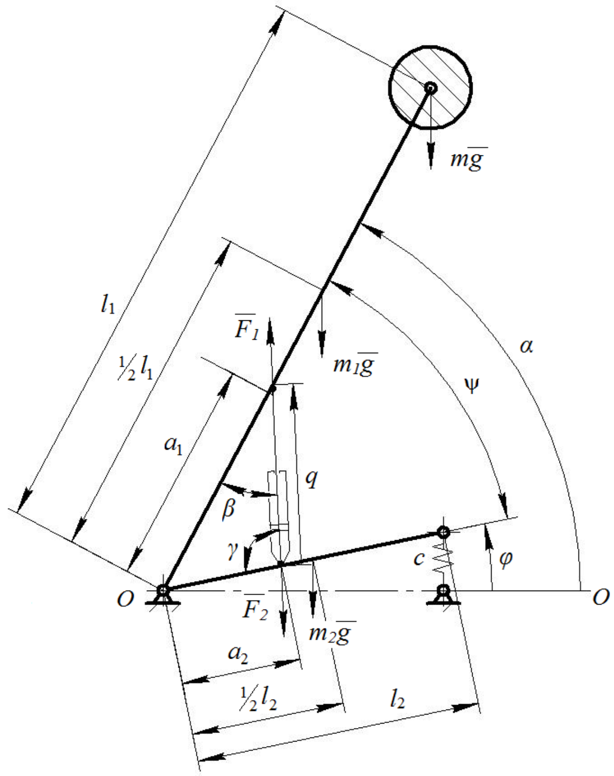

Fig. 1. Dynamic model of the manipulator boom system mounted on an elastic base

The adopted dynamic model consists of rigid boom links with a length of l_1 and a main frame with a length of l_2. The main frame has two support points, one of which is shared with the support joint of the boom and is assumed to be completely rigid, and the other is replaced by a movable link that reflects the elastic properties of the deformed support mechanism. A movable elastic yoke have a characterized by a combined elasticity coefficient *c*. The drive mechanism in the form of a hydraulic cylinder placed between the boom and the support frame and attached to the boom at a distance a_1 and to the support frame at a distance a_2, which are measured from the rigid support of the boom (the pivot point of the boom).

ISSN(online)2709-6149. Mining, constructional, road and melioration machines, 101, 2023, 11-20

In the adopted dynamic model of the manipulator, it is assumed that the load is mass m concentrated at the end of the boom and rigidly attached to its base. It is center of mass of the boom m_1 and the supporting frame m_2 placed in the middle of the corresponding elements and regardless of the geometry of the specified parts are at distances $\frac{1}{2}l_1$ and $\frac{1}{2}l_2$ from the boom's support hinge. Friction and backlash in kinematic pairs was not take into account, and the mechanical system of the boom was placed on a horizontal supporting surface.

The angle φ of rotation of the support frame and the angle α of rotation of the boom with respect to the horizontal surface are taken as generalized independent coordinates. The angle of rotation of the support frame will be determined by the deformation of the elastic element of the support frame. The movement of the rod of the drive hydraulic cylinder will determine the change an angle of rotation of the boom.

Change the departure of the load is performed by the manipulator in an absolutely unchanged space due to the joint movement of the boom relative to the support frame and vibrations of the support frame relative to the horizontal support surface, i.e.

$$\alpha = \psi + \varphi. \quad (1)$$

Within the framework of Newton's theory, a mathematical model of the considered holonomic beam system was determined using Lagrange equations of the second kind [18, 19], which is written in the form of a system of recurrent differential equations [7]

$$\begin{cases} (J_1 + ml_1^2)\ddot{\alpha} = M - (m + \frac{m_1}{2})gl_1\cos\alpha; \\ J_2\ddot{\varphi} = -M - m_2gl_2\cos\varphi - cl_2^2(\varphi - \varphi_0), \end{cases} \quad (2)$$

where:

$J_1 = m_1l_1^2/3$ and $J_2 = m_2l_2^2/3$ is moments of inertia of the boom and the support frame relative to their turning points;

$\ddot{\alpha}$ and $\ddot{\varphi}$ is an angular accelerations of the boom and support on top;

M is the external driving torque of the drive;

φ_0 is the initial deviation of the support link (in the following it is accepted $\varphi_0 = 0$).

The first and second equations of system (2) are connected by a moment determined by the following relationship

$$M = F_1a_1a_2\frac{\sin(\alpha - \varphi)}{\sqrt{a_1^2 + a_2^2 - 2a_1a_2\cos(\alpha - \varphi)}}, \quad (3)$$

where:

F_1 is forces on the rod of the hydraulic cylinder.

From the first equation of system (2), the driving moment M is determined, which depends on the coordinate α and has the next view

$$\frac{d^2M}{dt^2} = \ddot{M} = (J_1 + ml_1^2)\overset{IV}{\alpha} - \\ -(m + \frac{m_1}{2})gl_1(\ddot{\alpha}\sin\alpha + \dot{\alpha}^2\cos\alpha). \quad (5)$$

Let's mark

$$f = \left(\overset{IV}{\alpha} - k^2(\ddot{\alpha}\sin\alpha + \dot{\alpha}^2\cos\alpha)\right)^2, \quad (6)$$

where:

$$k = \sqrt{\frac{(m + \frac{m_1}{2})gl_1}{(J_1 + ml_1^2)}}.$$

Let's determine the equation of the criterion for estimating the minimum of the root mean square value of the change in the acceleration of the driving torque

$$\ddot{M}_{ck} = \sqrt{\int_0^{t_1}\ddot{M}^2dt} \to \min \quad (7)$$

or taking into account the expression (6), the set problem is equivalent to the minimization of the functional

$$\frac{1}{(J_1+ml_1^2)}\sqrt{\int_0^{t_1} f dt} \to \min \qquad (8)$$

From equation (8), it is obvious that the minimum value of the function will be provide that the integrand acquires the minimum value.

Let's solve the set variational problem (8) by numerical methods, ensuring the following boundary conditions of motion:

$$\begin{aligned} &t=0:\alpha=\alpha_0, \dot\alpha=0, \ddot\alpha=0, \dddot\alpha=0;\\ &t=t_1:\alpha=\alpha_к, \dot\alpha=0, \ddot\alpha=0, \dddot\alpha=0. \end{aligned} \qquad (9)$$

The condition for the minimum of the functional (8) is the fourth-order Poisson equation [7, 20]:

$$\frac{\partial f}{\partial\alpha}+\sum_{n=1}^{4}(-1)^n\frac{d^n}{dt^n}\frac{\partial f}{\partial\alpha^{(n)}}=0. \qquad (10)$$

It was find the derivatives of equation (10) from expression (8);

$$\begin{aligned} \frac{\partial f}{\partial\alpha} &= -2k^2\left(\ddot\alpha\cos\alpha-\dot\alpha^2\sin\alpha\right)\times\\ &\times\left(\overset{IV}{\alpha}-k^2(\ddot\alpha\sin\alpha+\dot\alpha^2\cos\alpha)\right); \end{aligned} \qquad (11)$$

$$\begin{aligned} \frac{\partial f}{\partial\dot\alpha} &= -4k^2\cos\alpha\dot\alpha\times\\ &\times\left(\overset{IV}{\alpha}-k^2(\ddot\alpha\sin\alpha+\dot\alpha^2\cos\alpha)\right); \end{aligned} \qquad (12)$$

$$\begin{aligned} \frac{d}{dt}\frac{\partial f}{\partial\dot\alpha} &= -2k^2[2k^2\sin 2\alpha\dot\alpha^4+2\cos\alpha\ddot\alpha\times\\ &\times(\overset{IV}{\alpha}-k^2\ddot\alpha\sin\alpha)-\dot\alpha^2(k^2(3+5\cos 2\alpha)\ddot\alpha+\\ &+2\sin\alpha\overset{IV}{\alpha})+2\cos\alpha\dot\alpha(\overset{V}{\alpha}-k^2\sin\alpha\dddot\alpha)]; \end{aligned} \qquad (13)$$

$$\frac{\partial f}{\partial\ddot\alpha}=2k^2\sin\alpha\left(k^2(\ddot\alpha\sin\alpha+\dot\alpha^2\cos\alpha)-\overset{IV}{\alpha}\right); \qquad (14)$$

$$\begin{aligned} \frac{d}{dt}\frac{\partial f}{\partial\ddot\alpha} &= 2k^2[k^2\cos 2\alpha\dot\alpha^3+\cos\alpha\dot\alpha\times\\ &\times(4k^2\ddot\alpha\sin\alpha-\overset{IV}{\alpha})+\sin\alpha(k^2\sin\alpha\dddot\alpha-\overset{V}{\alpha})]; \end{aligned} \qquad (15)$$

$$\begin{aligned} \frac{d^2}{dt^2}\frac{\partial f}{\partial\ddot\alpha} &= k^2[2k^2\sin 2\alpha(-2\dot\alpha^4+2\ddot\alpha^2+\\ &+3\dot\alpha\dddot\alpha)+k^2\cos 2\alpha(14\dot\alpha^2\ddot\alpha-\overset{IV}{\alpha})+k^2\overset{IV}{\alpha}-\\ &-2\cos\alpha(\ddot\alpha\overset{IV}{\alpha}+2\dot\alpha\overset{V}{\alpha})+2\sin\alpha(\dot\alpha^2\overset{IV}{\alpha}-\overset{VI}{\alpha})]; \end{aligned} \qquad (16)$$

$$\frac{\partial f}{\partial\dddot\alpha}=0;\ \frac{d}{dt}\frac{\partial f}{\partial\dddot\alpha}=0; \qquad (17)$$

$$\frac{d^2}{dt^2}\frac{\partial f}{\partial\dddot\alpha}=0;\ \frac{d^3}{dt^3}\frac{\partial f}{\partial\dddot\alpha}=0; \qquad (18)$$

$$\frac{\partial f}{\partial\overset{IV}{\alpha}}=2[\overset{IV}{\alpha}-k^2(\cos\alpha\dot\alpha^2+\sin\alpha\ddot\alpha)]; \qquad (19)$$

$$\begin{aligned} \frac{d}{dt}\frac{\partial f}{\partial\overset{IV}{\alpha}} &= 2[k^2(-3\cos\alpha\dot\alpha\ddot\alpha+\sin\alpha\times\\ &\times(\dot\alpha^3-\dddot\alpha))+\overset{V}{\alpha}]; \end{aligned} \qquad (20)$$

$$\begin{aligned} \frac{d^2}{dt^2}\frac{\partial f}{\partial\overset{IV}{\alpha}} &= 2[k^2(\cos\alpha(\dot\alpha^4-3\ddot\alpha^2-4\dot\alpha\dddot\alpha)+\\ &+\sin\alpha(6\dot\alpha^2\ddot\alpha-\overset{IV}{\alpha}))+\overset{VI}{\alpha}]; \end{aligned} \qquad (21)$$

$$\begin{aligned} \frac{d^3}{dt^3}\frac{\partial f}{\partial\overset{IV}{\alpha}} &= 2[k^2(5\cos\alpha(2\ddot\alpha(\dot\alpha^3-\dddot\alpha)-\dot\alpha\overset{IV}{\alpha})-\\ &-\sin\alpha(\dot\alpha^5-15\dot\alpha\ddot\alpha^2-10\dot\alpha^2\dddot\alpha+\overset{V}{\alpha}))+\overset{VII}{\alpha}]; \end{aligned} \qquad (22)$$

$$\begin{aligned} \frac{d^4}{dt^4}\frac{\partial f}{\partial\overset{IV}{\alpha}} &= -2k^2\cos\alpha[\dot\alpha^6-45\dot\alpha^2\ddot\alpha^2-\\ &-20\dot\alpha^3\dddot\alpha+10\dddot\alpha^2+15\ddot\alpha\overset{IV}{\alpha}+6\dot\alpha\overset{V}{\alpha})-\\ &-2k^2\sin\alpha(-15(-\dot\alpha^4\ddot\alpha+\ddot\alpha^3+4\dot\alpha\ddot\alpha\dddot\alpha+\\ &+\dot\alpha^2\overset{IV}{\alpha})+\overset{VI}{\alpha}]+2\overset{VII}{\alpha}; \end{aligned} \qquad (23)$$

After substituting expressions (11)…(23) into equation (10), a complex nonlinear differential equation of the 8th order is obtained for the solution of which we will apply the Runge-Kutta numerical method:

$$[-k^2\cos\alpha\dot{\alpha}^6+3k^4\cos\alpha\sin\alpha\ddot{\alpha}^2+$$
$$+15k^2\sin\alpha\ddot{\alpha}^3-k^2\sin\alpha\dot{\alpha}^4\times$$
$$\times(k^2\cos\alpha+15\ddot{\alpha})+20k^2\cos\alpha\dot{\alpha}^3\dddot{\alpha}-$$
$$-10k^2\cos\alpha\dddot{\alpha}^2+k^4\sin^2\alpha\overset{IV}{\alpha}-$$
$$-15k^2\cos\alpha\ddot{\alpha}\overset{IV}{\alpha}+3k^2\dot{\alpha}^2\times \tag{24}$$
$$\times(-2k^2\sin^2\alpha\ddot{\alpha}+15\cos\alpha\ddot{\alpha}^2+$$
$$+5\sin\alpha\overset{IV}{\alpha})+2k^2\dot{\alpha}(2\sin\alpha\times$$
$$\times(k^2\cos\alpha+15\ddot{\alpha})\dddot{\alpha}-$$
$$-3\cos\alpha\overset{V}{\alpha})-2k^2\sin\alpha\overset{VI}{\alpha}+\overset{VIII}{\alpha}]=0.$$

Have been find a solution to the given problem in the section of the arrow movement from 45° to 85° for a duration of movement of 3 s. The parameters of the mechanical system it was adopt as follows: $m_1=300$ kg; $m_2=100$ kg; $m=900$ kg; $l_1=4$ meter; $l_2=2$ meter; c = 500 000 H/m^2; α_0 = 0,8 rad. (45°); α_k = 1,45 rad. (85°).

On Fig. 2 *a*...*d* shows the graphs of the kinematic characteristics of the boom, which reflect the solution of the problem (24) under the given boundary conditions (9). The results of the found numerical solution of equation (24) were substituted into system (2), from which the dependences of the change in the angle φ(t) of rotation of the support frame of the manipulator relative to the horizontal surface were found (see Fig. 2 *d*).

We compare the obtained numerical solutions of equation (24) with the integral curves of the following linear differential equation of the 8th order:

$$\overset{VIII}{\alpha_1}=0. \tag{25}$$

The solution of equation (25) is taken as a polynomial of the 7th degree:

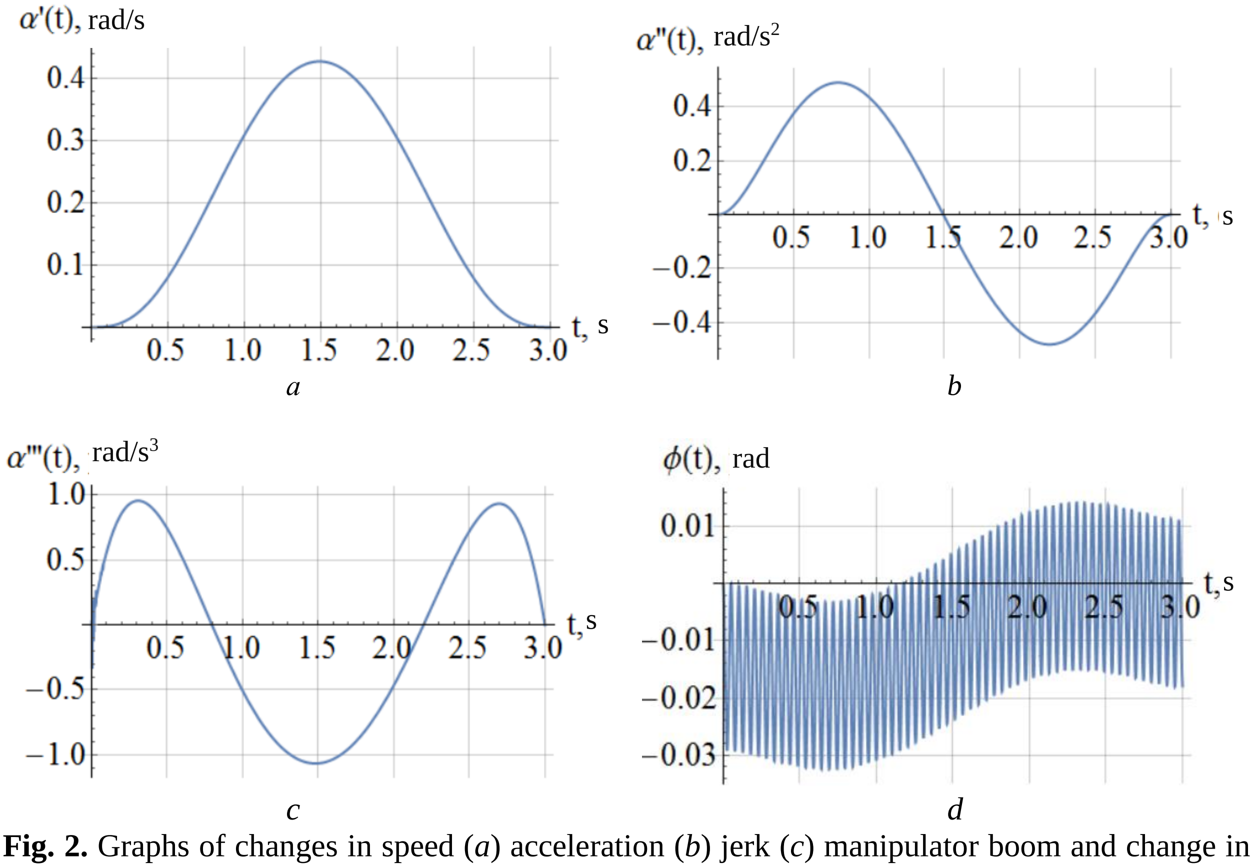

Fig. 2. Graphs of changes in speed (*a*) acceleration (*b*) jerk (*c*) manipulator boom and change in the angle of rotation of the supporting frame (*d*) relative to the horizontal surface

$$\alpha_1(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 + + C_5 t^5 + C_6 t^6 + C_7 t^7, \; 0 \le t \le t_1, \quad (26)$$

where:
C_0, C_1, ... C_7 are constants of integration determined by given boundary conditions (9).

The first, second, and third derivatives of expression (26) were determined:

$$\dot{\alpha}_1(t) = C_1 + 2C_2 t + 3C_3 t^2 + 4C_4 t^3 + + 5C_5 t^4 + 6C_6 t^5 + 7C_7 t^6. \quad (27)$$

$$\ddot{\alpha}_1(t) = 2C_2 + 6C_3 t + 12C_4 t^2 + 20C_5 t^3 + + 30C_6 t^4 + 42C_7 t^5. \quad (28)$$

$$\dddot{\alpha}_1(t) = 6C_3 + 24C_4 t + 60C_5 t^2 + + 120C_6 t^3 + 210C_7 t^4. \quad (29)$$

After substituting the initial conditions at $t = 0$, constant coefficients were found from expression (9) in equations (26)...(29):

$$C_0 = \alpha_0, \; C_1 = 0, \; C_2 = 0, \; C_3 = 0. \quad (30)$$

As a result of substituting the final conditions at $t = t_1$ from equality (9) and constant integrations (30) in dependence (26)…(29), we obtain a system of linear equations

$$\begin{cases} C_4 + C_5 t_1 + C_6 t_1^2 + C_7 t_1^3 = \dfrac{\alpha_к - \alpha_0}{t_1^4}; \\ 4C_4 + 5C_5 t_1 + 6C_6 t_1^2 + 7C_7 t_1^3 = 0; \\ 12C_4 + 20C_5 t_1 + 30C_6 t_1^2 + 42C_7 t_1^3 = 0; \\ 24C_4 + 60C_5 t_1 + 120C_6 t_1^2 + 210C_7 t_1^3 = 0. \end{cases} \quad (31)$$

The solution of the system of equations (31) gives the following values of constant coefficients

$$C_4 = 35\frac{\alpha_k - \alpha_0}{t_1^4}, \; C_5 = -84\frac{\alpha_k - \alpha_0}{t_1^5}, \quad (32)$$

$$C_6 = 70\frac{\alpha_k - \alpha_0}{t_1^6}; \; C_7 = -20\frac{\alpha_k - \alpha_0}{t_1^7}. \quad (33)$$

In fig. 3 shows a comparison of graphs of the obtained solutions of equations (24) and (25), from which it is clear that in dependence (24) the dominant component that determines its solution is the term with the highest 8th degree of the derivative. Thus, the approximate solution of equation (24) can be express in the form of polynomial (26). For the exact solution of the solution of the given variational problem (24), it is possible to apply a complex polynomial, which consists of two terms.

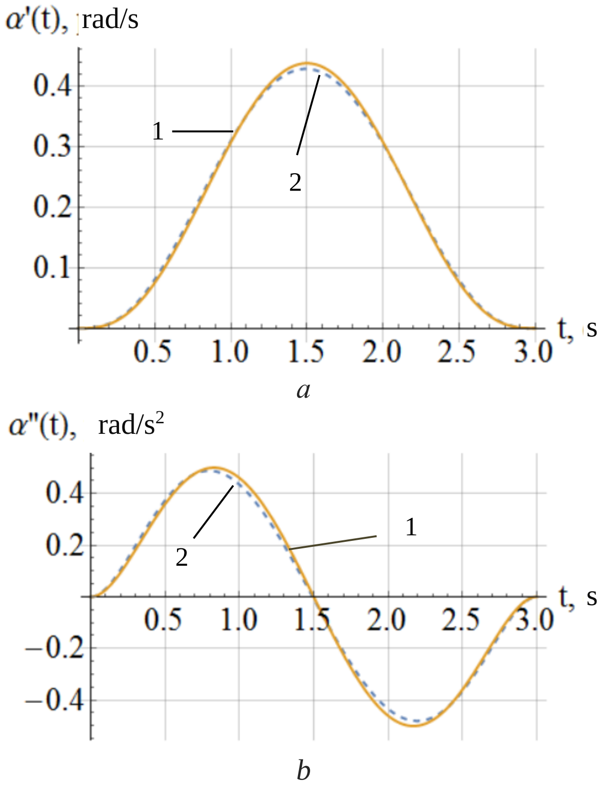

Fig. 3. Graphs illustrating the change in kinematic and dynamic characteristics of the system over time: the function of the angular velocity α (a) and its acceleration (b); *1* – for equation (25); *2* – for equation (24)

$$\alpha(t) = \alpha_1(t) + \alpha_2(t), \; 0 \le t \le t_1. \quad (34)$$

In equation (34), the first term $\alpha_1(t)$ is a polynomial selected from the condition of ensuring the boundary conditions of motion (9), and the second term $\alpha_2(t)$ is a polynomial that determines the free coefficients and satisfies such boundary conditions

$$\alpha_2(0) = 0, \dot{\alpha}_2(0) = 0, \ddot{\alpha}_2(0) = 0; \dddot{\alpha}_2(0) = 0; \alpha_2(t_1) = 0, \dot{\alpha}_2(t_1) = 0, \ddot{\alpha}_2(t_1) = 0; \dddot{\alpha}_2(t_1) = 0. \quad (35)$$

The polynomial $\alpha_2(t)$ must provide the minimum of the integral functional (8), so it can be used in the following form:

$$\alpha_2(t) = t^3(t - t_1)^3(A_0 + \ldots + A_n t^n), \quad (36)$$

where: $0 \le t \le t_1$.

The multiplier $t^3(t-t_1)^3$ in expression (36) guarantees the fulfillment of the target boundary conditions for any coefficients $A_0, \ldots A_n$. These coefficients are free and used to find the minimum of the functional (8).

The effectiveness of the application of the optimal criterion of the rms value of the acceleration of the drive torque with the optimal criterion of the rms value of the speed of change of the driving torque of the motion modes for the boom system of the manipulator under consideration is compared. Fig. 4 show the graphs that shows the nature and amplitudes of changes in the oscillations of the manipulator support link for the indicated optimal modes of movement of the manipulator boom.

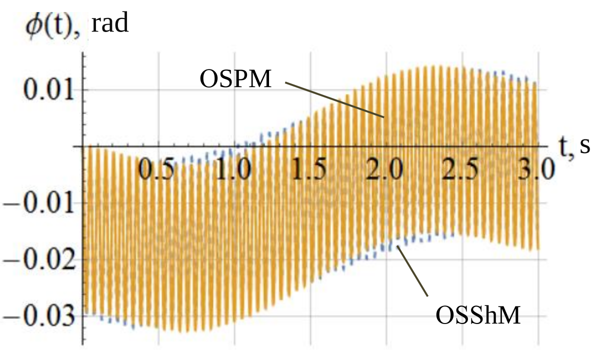

Fig. 4. Comparison of the optimal according to the criterion of the rms value of the acceleration of the drive driving torque (OSPM) and the optimal according to the criterion of the rms value of the rate of change of the torque (OSShM) of the driving modes

The results of the calculations are given in the table below.

From the above table of the results of the comparison of the oscillations of the rotation angle of the manipulator support frame for optimal according to the criteria of the root mean square value of the speed and acceleration of the change of the drive torque, it can be seen that the deviation between the maximum values of the amplitude of the oscillations can be in the range from 0,3% to 19% in different time periods. At the same time, the maximum value of the angle oscillation amplitude does not exceed 0,03 radians, i.e. micro oscillations of the elastic support mechanism of the manipulator are observed.

Table. Comparison of values of the angle function turn $\varphi(t)$ for different driving modes

t, s	$\varphi(t)$, rad.	
	OSPM	OSShM
0	0	0
0,5	-0,017672	-0,017731
1	-0,028897	-0,030872
1,5	-0,008129	-0,008277
2	0,010377	0,012374
2,5	-0,002857	-0,002471
3	-0,017877	-0,01789

DISCUSSION OF THE RESULTS

The analysis of the obtained results of the calculation of the optimum according to the criterion of the mean square value of the change in the acceleration of the driving moment of the movement mode of the boom system of the manipulator, which is installed on an elastic base, shows that an increase in the smoothness of the change in the acceleration of the change in the angular coordinate of the boom significantly reduces the oscillatory processes that occur during its movement [7, 16, 20].

The obtained solution of the optimal motion mode can be replaced by an integral polynomial of the differential equation of the 8th order, which greatly simplifies the calculation of such a motion mode.

A comparison of the optimal modes of movement according to the criteria of the mean square value of the change in speed and the acceleration of the drive torque showed that the smoothness of the change in the acceleration of the angular coordinate of the boom has a significant effect on the system dynamics, which can be achieved when optimizing according to the criterion of the mean square value of the change in the speed of the driving torque, and optimization according to higher derivatives from the function of changing the driving moment, the boom dynamics are not changed significantly.

The decision of the optimal mode of movement of the manipulator based on the criterion of the root mean square value of the change in acceleration was apply to calculate the driving torque of the drive (4) and its rate of change over time. The calculation results shown in Fig. 5.

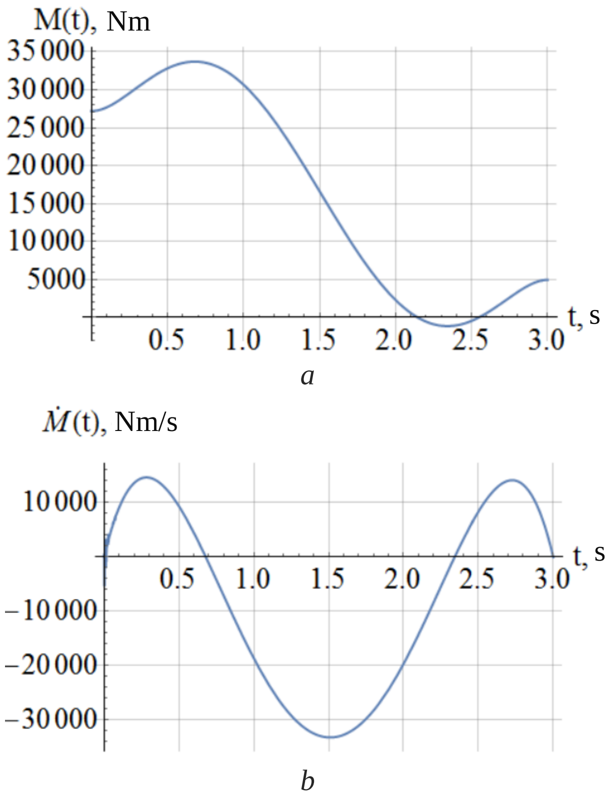

Fig. 5. Graphs illustrating the change in the function of the driving torque *M*(t) (*a*) and the rate of change of the torque over time (*b*)

On Fig. 6 shows the graph of the comparison of the oscillations of the support mechanism of the manipulator, determined according to the criteria of the minimum root mean square values of the driving torque [7] and the acceleration of the change in time of the driving torque of the drive. The graphs of the dependences show that the amplitude of oscillations in the mode according to the criterion of the root mean square value of the acceleration of the driving torque change is almost always smaller than the amplitude of oscillations in the mode according to the criterion of the root mean square value of the drive torque. At the same time, the deviation of the amplitude of oscillations for the obtained optimal mode of movement from the amplitude of oscillations determined by the criterion of the root mean square value of the drive torque [7] is 40% for the maximum value and 60% for the average values. This indicates the advantages of using the optimal mode of movement according to the criterion of the root mean square value of the acceleration of the change in the driving torque of the drive mechanism.

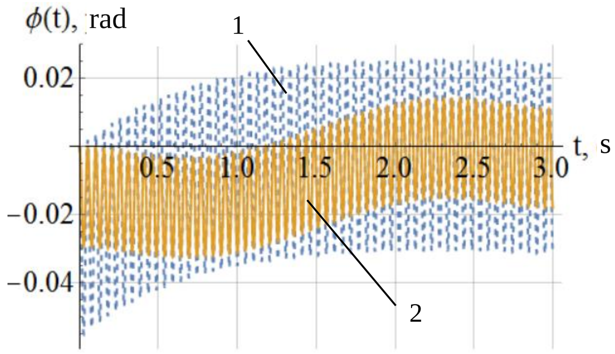

Fig. 6. Oscillation graphs of the support mechanism of the manipulator for the minimum mean square value of the driving moment of the drive (*1*) and the mean square acceleration of the driving moment of the drive (*2*) optimal according to the criterion

CONCLUSIONS

Optimization of the manipulator movement mode based on the criterion of the mean square value of the acceleration of the change of the driving torque allows to achieve a smooth movement mode of the manipulator boom, which is evidenced by the figures shown in Fig. 4 graphical dependencies. The change in the angular coordinate of rotation of the support mechanism of the manipulator for the obtained mode of movement will be carried out with micro-oscillations of insignificant amplitude.

It was established that the conditions for ensuring the boundary conditions of motion play a dominant role in the minimization of the integral criterion. Moreover, it is possible to hypothesize that it is the values of the boundary conditions and their number that provide the value of the optimization criterion.

Note that the application of motion modes obtained on the basis of the derivative of the driving moment allows to increase the smoothness of the movement of the boom system of the manipulator, and therefore to reduce the amplitude of oscillations of the support mechanism of the manipulator.

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Дослідження оптимального режиму руху маніпулятора робота на пружній опорі за критерієм середньоквадратичного значення прискорення рушійного моменту приводу

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Анотація. Попередніми дослідженнями оптимізації режиму руху маніпулятора на пружній основі за критеріями середньоквадратичного значення рушійного моменту приводу та швидкості зміни середньоквадратичного значення моменту приводу було встановлено, що застосування оптимальних режимів руху маніпулятора, які створені на основі вищих похідних від характеристики зміни рушійного моменту, дозволяють отримати більш плавні переміщення стрілової системи маніпулятора, тим самим зменшити його коливання на пружній опорі. При цьому, отримані оптимальні режими руху повністю не усувають коливання стрілової системи в процесі зміни вильоту. За спостереженням коливань пружної ланки маніпулятора було оцінено якість отриманих оптимальних режимів руху та показано наявність залишкових коливань.

В даному дослідженні розглядається можливість оптимізації режиму руху маніпулятора за критерієм середньоквадратичного значення прискорення зміни рушійного моменту приводу. Для цього, спочатку було визначено другу похідну від функції рушійного моменту приводу стрілової системи маніпулятора. Такий критерій оптимізації режиму руху представляє собою інтегральний функціонал, мінімізація якого здійснена методами варіаційного числення. Закономірність зміни приводного моменту знайдено з динамічного рівняння руху.

Пошук оптимального режиму руху маніпулятора реалізується через розв'язок краєвого нелінійного диференційного рівняння Пуассона четвертого порядку.

Результати проведених досліджень дозволили значно краще зрозуніти природу коливань стрілової системи маніпулятора і, як наслідок, розробити систему керування приводом, яка дозволяє реалізувати отриманий оптимальний режим руху.

Ключові слова: маніпулятор, пружна основа, критерій оптимізації, середньоквадратичне значення прискорення зміни моменту, мінімізація коливань.