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Research on the change of the combined moment of inertia for two-link manipulator with angular coordinate system

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Abstract. The main of the problems of the development of two-link manipulators with an angular coordinate system is the lack of effective and universal control systems capable of implementing energy-intensive modes of movement. This is due to the design features of the pointer system of such manipulators and the methods of their use. During the operation of the manipulator, significant dynamic loads occur in its drive system, which create oscillations of the mechanical system and reduce the accuracy of work. The control system must also effectively compensate for such dynamic variations.

Dynamic equations of motion used to build manipulator control controllers, which allow taking into account various features of using manipulators. These mathematical models necessarily contain one of the characteristics in the form of the combined moment of inertia of the mechanical system, which is important in the calculation of differential equations and the determination of kinematic parameters of motion.

In this article, the combined moments of inertia of the boom system of a two-link manipulator with an angular coordinate system are determined. A study was carry out and it shown that the nature of the change in the combined moments of inertia influenced by the modes of movement of the boom system links.

The results of the study showed that the combined moment of inertia of the boom system of the manipulator is not a constant value, which does not change uniformly under special operating conditions of the machine.

To determine the combined moments of inertia, the method of bringing the kinetic energy of the moving system to the drive mechanism was apply. This is relevant, because in the future it will be necessary to adjust the unevenness of the movement of the manipulator links by the drive itself.

The study was conduct for a typical linear motion mode and a motion mode with a change in speed along a parabolic trajectory.

Keywords: two-link manipulator, the kinetic energy, combined moment of inertia, the dynamic model, equations of motion.

INTRODUCTION

Two-links manipulators have found wide application in various industries and robotic systems [1, 2]. In industrial production, two-link manipulators are used for the automation of production processes, ensuring fast and accurate performance of tasks, which allows increasing the productivity of various technological processes and reducing costs [3, 4]. In addition, such manipulators are widely used in robotics. Here they make it possible to create robots that can perform various tasks, such as assembly of products on production lines and processes of precision welding of parts [5, 6]. In the medical field, two-link manipulators are becoming auxiliary tools in surgical processes, providing precision and stability of movements, allowing doctors to perform complex manipulations with great precision and minimal risk for patients [7].

Thus, two-link manipulators are an integral part of many modern technological processes, and their application expands and changes according to the needs of society and the development of technologies.

One of the important tasks in engineering is the design of efficient technical systems with low energy consumption [8, 9]. To implement such tasks, it is necessary to have an accurate idea of the dynamic processes that occur in the component parts of machines [10, 11]. In particular, the dynamics of the movement modes of the manipulator boom will significantly affect the efficiency of the energy supply of the drive of such a technical system [12-14]. The main keys parameters of changing dynamic loads in the manipulator drive system is the value of its combined moment of inertia of the boom [15]. This is because this parameter for a two-link manipulator is a variable characteristic, which depends not only on the mass-geometric characteristics of the links of its mechanical system, but also on their mutual location and position in space. Changing the configuration of the manipulator boom and the position of the component links in space will lead to a change in the combined moment of inertia.

There are not scientific studies that would show the nature of the change in the combined moment of inertia for a two-link manipulator. Therefore, this article will consider the specifics of determining such a characteristic and investigate the nature of its change.

PURPOSE OF THE ARTICLE

To develop a mathematical model for determining the combined moment of inertia of the boom of a two-link angular manipulator with two degrees of mobility and to investigate the nature of the change in this dependence.

PRESENTING MAIN MATERIAL

Consider a manipulator consisting of two rigid independently moving links 1 and 2 (see Fig. 1) with masses m_1 та m_2. All links interconnected by kinematic pairs of the 5-th class of the cylindrical hinge type [16]. The drive mechanisms of each of the links are located at the locations of the hinges and can develop torques by M_1 та M_2, (the actuators of the manipulator links were not show on the considered scheme) [17, 18].

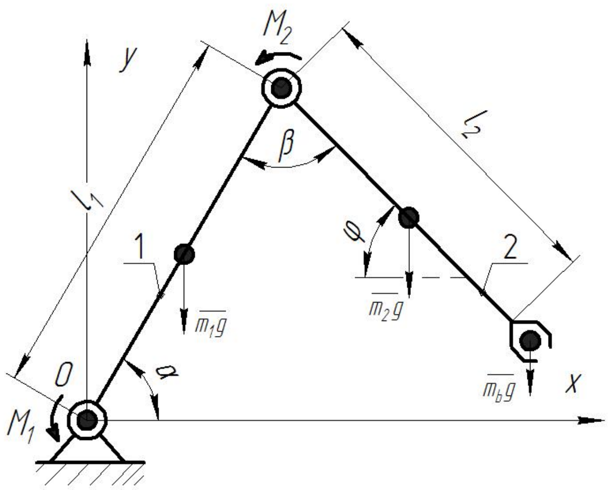

Fig. 1. Dynamic diagram of a two-link angular manipulator

The rotation angles α and β of the links were took as generalized coordinates in this scheme.

The equation of motion of each of the moving links of the manipulator will be determined on the basis of Lagrange equations of the 2nd kind, which in a simplified form will be as follows [19]:

$$\begin{cases} J_{\text{зв}1}\dfrac{d\omega_1}{dt} = M_1 - M_{c1}; \\ J_{\text{зв}2}\dfrac{d\omega_2}{dt} = M_2 - M_{c2}, \end{cases} \qquad (1)$$

where: M_{c1}, M_{c2} is static resistance moments; M_1, M_2 is driving moments; $J_{\text{зв}1}$, $J_{\text{зв}2}$ combine moments of inertia, respectively, for the first and second drive mechanisms of the manipulator; ω_1, ω_2 the angular velocities of manipulator links relative to the horizontal axis, i.e. $\omega_1 = d\alpha/dt$ and $\omega_2 = d\varphi/dt$.

From the system of equations (1), it is obvious that the driving moments for each of the drive links will be determined as the total of static and dynamic moments, i.e.:

$$\begin{cases} M_1 = J_{\text{зв}1}\dfrac{d\omega_1}{dt} + M_{c1}; \\ M_2 = J_{\text{зв}2}\dfrac{d\omega_2}{dt} + M_{c2}. \end{cases} \qquad (2)$$

From the system of equations, it is obvious that the component of the dynamic moment of the drive depends on the combined moment of inertia and angular acceleration of the rotary link. As is known, the static moment of the manipulator's boom will be determine only by the position of the links.

It is know that for moment starting and braking in the boom system of the manipulator, significant loads from inertial forces occur, which leads to oscillations of the moving links of the manipulator [10, 11, and 12]. This significantly reduces the energy efficiency of using the manipulator in transient modes of its movement.

It was determine the dependencies of the combined moments of inertia for each of the drive links.

The equation of the kinetic energy of the manipulator reduced to the first drive mechanism [19]:

$$T_1 = \frac{J_{зв1}\omega_{зв1}^2}{2}, \quad (3)$$

where: $\omega_{зв1}$ the angular velocity of the reduced link, which in this scheme will be equal to the angular velocity of the first link ω_1.

The full kinetic energy of the manipulator, expressed through its moving links, will be equal to:

$$T_1 = \frac{J_1\omega_1^2}{2} + \frac{J_{2c}\omega_2^2}{2} + \frac{m_2\upsilon_{2c}^2}{2} + \frac{m_b\upsilon_b^2}{2}, \quad (4)$$

where: m_b, υ_b respectively, mass and speed of movement of cargo; υ_{2c} is velocity of the center of mass of the link *2*, which simultaneously performs rotary and plane-parallel motion; $J_{2c} = \frac{m_2 l_2^2}{12}$ the moment of inertia of the link *2* relative to its center of mass; $J_1 = \frac{m_1 l_1^2}{3}$ the moment of inertia of link 1 relative to the point of its rotation.

By equating expressions (3) and (4), the combined moment of inertia of the manipulator boom, which reduced to the drive M_1, were determinate:

$$J_{зв1} = J_1 + \frac{J_{2c}\omega_2^2}{\omega_1^2} + \frac{m_2\upsilon_{2c}^2}{\omega_1^2} + \frac{m_b\upsilon_b^2}{\omega_1^2}. \quad (5)$$

The square of the speed of movement of the center of mass of link *2* was determinate through the component coordinates, i.e.:

$$\upsilon_{2c}^2 = \dot{x}_{2c}^2 + \dot{y}_{2c}^2, \quad (6)$$

where: $\dot{x}_{2c} = \frac{dx_{2c}}{dt}$, $\dot{y}_{2c} = \frac{dy_{2c}}{dt}$.

The coordinates of the center mass of link *2*:

$$x_{2c} = l_1\cos\alpha + \frac{1}{2} l_2 \cos\varphi, \quad (7)$$

$$y_{2c} = l_1\sin\alpha - \frac{1}{2} l_2 \sin\varphi. \quad (8)$$

Derivatives of expressions (7) and (8) were determinate:

$$\frac{dx_{2c}}{dt} = -l_1\dot{\alpha}\sin\alpha - \frac{1}{2}\dot{\varphi} l_2 \sin\varphi. \quad (9)$$

$$\frac{dy_{2c}}{dt} = l_1\dot{\alpha}\cos\alpha - \frac{1}{2}\dot{\varphi} l_2 \cos\varphi. \quad (10)$$

The squares of the velocities (9) and (10) will have the following form:

$$\left(\frac{dx_{2c}}{dt}\right)^2 = l_1^2\dot{\alpha}^2\sin^2\alpha + + l_1 l_2\dot{\alpha}\dot{\varphi}\sin\alpha\sin\varphi + \frac{1}{4}\dot{\varphi}^2 l_2^2 \sin^2\varphi. \quad (11)$$

$$\left(\frac{dy_{2c}}{dt}\right)^2 = l_1^2\dot{\alpha}^2\cos^2\alpha - - l_1 l_2\dot{\alpha}\dot{\varphi}\cos\alpha\cos\varphi + \frac{1}{4}\dot{\varphi}^2 l_2^2 \cos^2\varphi. \quad (12)$$

Considering the obtained expressions (11) and (12), the speed of the center of mass of link 2 will be equal to:

$$\upsilon_{2c}^2 = l_1^2\dot{\alpha}^2 + \frac{1}{4}\dot{\varphi}^2 l_2^2 - l_1 l_2 \dot{\alpha}\dot{\varphi}\cos(\alpha+\varphi). \quad (13)$$

Similarly, the square of the speed of the center of mass of the load was determinate:

$$\upsilon_b^2 = l_1^2\dot{\alpha}^2 + \dot{\varphi}^2 l_2^2 - 2l_1 l_2 \dot{\alpha}\dot{\varphi}\cos(\alpha+\varphi). \quad (14)$$

The ratio between the angular velocities of links 1 and 2 can be find through derivatives in generalized coordinates, i.e.:

$$\frac{\omega_2}{\omega_1} = \frac{\dot{\varphi}}{\dot{\alpha}}. \quad (15)$$

Taking into account that:

$$\varphi = \pi - \alpha - \beta \quad (16)$$

and

$$\dot{\varphi} = -\dot{\alpha} - \dot{\beta}, \quad (17)$$

then

$$\left(\frac{\omega_2}{\omega_1}\right)^2 = 1 + 2\frac{\dot{\beta}}{\dot{\alpha}} + \left(\frac{\dot{\beta}}{\dot{\alpha}}\right)^2. \quad (18)$$

$$\frac{\upsilon_{2c}^2}{\omega_1^2} = \frac{l_1^2\dot{\alpha}^2 + \frac{1}{4}\dot{\varphi}^2 l_2^2 - l_1 l_2\dot{\alpha}\dot{\varphi}\cos(\alpha+\varphi)}{\dot{\alpha}^2} = \quad (19)$$
$$= l_1^2 + \frac{l_2^2}{4}\frac{\dot{\varphi}^2}{\dot{\alpha}^2} - l_1 l_2 \frac{\dot{\varphi}}{\dot{\alpha}}\cos(\alpha+\varphi).$$

Similarly, it was find for cargo:

$$\frac{\upsilon_b^2}{\omega_1^2} = l_1^2 + \frac{\dot{\varphi}^2}{\dot{\alpha}^2} l_2^2 - 2l_1 l_2 \frac{\dot{\varphi}}{\dot{\alpha}}\cos(\alpha+\varphi). \quad (20)$$

Taking into expression the found expressions (18), (19) and (20), the moment of inertia of the manipulator arm reduced to the first drive mechanism it was determinate:

$$J_{зв1} = (J_1 + m_2 l_1^2 + m_b l_1^2) +$$
$$+ \left(J_{2c} + \frac{m_2 l_2^2}{4} + m_b l_2^2\right)\left(1 + 2\frac{\dot{\beta}}{\dot{\alpha}} + \left(\frac{\dot{\beta}}{\dot{\alpha}}\right)^2\right) + \quad (21)$$
$$- (m_2 l_1 l_2 + 2m_b l_1 l_2)\left(1 + \frac{\dot{\beta}}{\dot{\alpha}}\right)\cos\beta.$$

To determine the combined moment of inertia of the boom of the two-link manipulator, the equation of the kinetic energy of the manipulator, which reduced to the second drive mechanism, it was find:

$$T_2 = \frac{J_{зв2}\omega_{зв2}^2}{2}, \quad (22)$$

where: $\omega_{зв2}$ the angular velocity of the reduced link, which in this scheme will be equal to the angular velocity of the second link ω_2.

The kinetic energy of the second moving link of the manipulator with the load will be equal to:

$$T_2 = \frac{J_{2c}\omega_2^2}{2} + \frac{m_2\upsilon_{2c}^2}{2} + \frac{m_b\upsilon_b^2}{2}. \quad (23)$$

Taking into account (22) and (23), the combined moment of inertia of the second link of the manipulator was find, which reduced to the drive M_2:

$$J_{зв2} = J_{2c} + \frac{m_2\upsilon_{2c}^2}{\omega_2^2} + \frac{m_b\upsilon_b^2}{\omega_2^2}, \quad (24)$$

where:

$$\frac{\upsilon_{2c}^2}{\omega_2^2} = \frac{l_1^2\dot{\alpha}^2 + \frac{1}{4}\dot{\varphi}^2 l_2^2 - l_1 l_2\dot{\alpha}\dot{\varphi}\cos(\alpha+\varphi)}{\dot{\varphi}^2} = \quad (25)$$
$$= l_1^2\frac{\dot{\alpha}^2}{\dot{\varphi}^2} + \frac{l_2^2}{4} - l_1 l_2\frac{\dot{\alpha}}{\dot{\varphi}}\cos(\alpha+\varphi);$$

$$\frac{\upsilon_b^2}{\omega_1^2} = l_1^2\frac{\dot{\alpha}^2}{\dot{\varphi}^2} + l_2^2 - 2l_1 l_2\frac{\dot{\alpha}}{\dot{\varphi}}\cos(\alpha+\varphi). \quad (26)$$

Taking into expression (25) and (26), the combined moment of inertia for the link *2* is finally determined:

$$J_{зв2} = J_{\Sigma 2} + (m_2 + m_b) l_1^2 \frac{\dot{\alpha}^2}{(\dot{\alpha}+\dot{\beta})^2} + \quad (27)$$
$$- (m_2 + 2m_b) l_1 l_2 \frac{\dot{\alpha}}{(\dot{\alpha}+\dot{\beta})}\cos(\beta),$$

where: $J_{\Sigma 2} = J_{2c} + \frac{m_2 l_2^2}{4} + m_b l_2^2$.

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As can seen from the obtained formulas (21) and (24), the value of the summary moments of inertia of the manipulator boom, in addition to the mass-geometric parameters of the boom links, also depends on the nature of the change in the angular velocities of the component links of the manipulator boom and the angle of mutual rotation between link *1* and *2*.

It was investigate the dependence of the change in the combined moments of inertia of the two-link manipulator boom for a typical linear mode of motion. The dependence of the change in the angular velocities of the links of the manipulator during its acceleration will be express by the following characteristics:

$$\dot{\alpha} = \frac{\omega_{1\_\max}}{t_1} t\,; \qquad (28)$$

$$\dot{\beta} = \frac{\omega_{2\_\max}}{t_1} t\,, \qquad (29)$$

where: $\omega_{1\_\max}$ and $\omega_{2\_\max}$ is a maximum link acceleration velocities by *1* and *2*; t_1 acceleration time.

For the specified motion modes (28) and (29), the dependences of changes in angular coordinates will be determinate by the next formula:

$$\alpha(t) = \alpha_0 + \dot{\alpha}\frac{t}{2}, \qquad (30)$$

where: α is a general coordinate of a mechanical system (α або β); α_0 the initial position of a generalized coordinate; $\dot{\alpha}$ the velocity of the corresponding coordinate, which is determined by (28) or (29).

In this work, the changes of the combined moments of inertia for the manipulator's mechanical system with the next parameters were investigate: $m_1 = m_2 = 200$ kg; $m_b = 800$ kg; $l_1 = l_2 = 2$ m.

Fig. 2 show the graphs of the functions with combined moments of inertia for a typical lin-

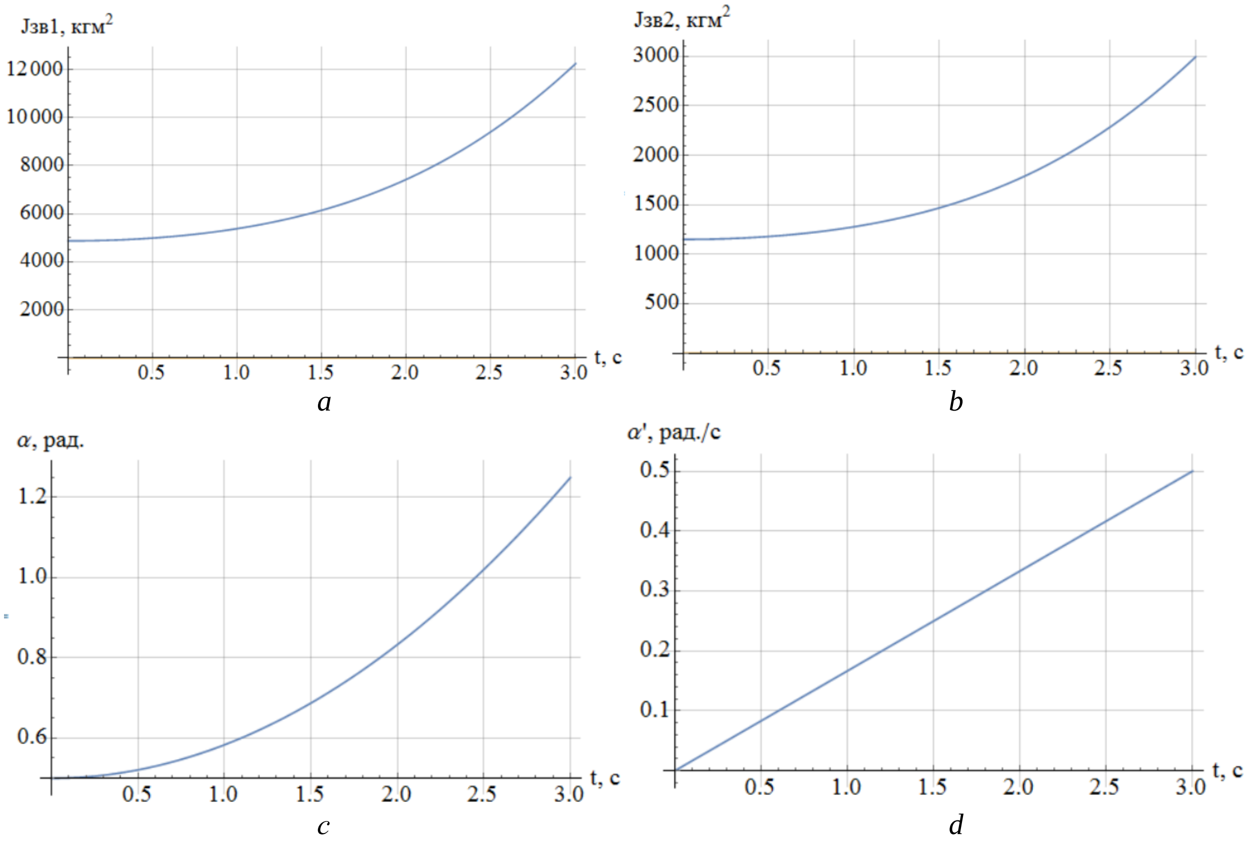

Fig. 2. Graphs of the functions of the combined moments of inertia for a typical mode of motion: *a* – dependency change $J_{зв1}$; *b* – dependency change $J_{зв2}$; *c* – dependence of the change in the angle of rotation α; *d* – dependence of the change in angular velocity dα/d*t*

ear mode of motion with the next parameters: t_1 = 3 s; $\omega_{1\_max} = \omega_{2\_max}$ = 0,5 rad./s; α_0 = 0,5 rad.; β_0 = 0,4 rad.

It can be seen from the presented graphs that in the process of moving the links of the manipulator, there is a change in the combined moments of inertia of the boom, in particular, for a given typical mode of movement, their increase occurs. This means that in the process of changing the boom position, the dynamic load component will increase.

The modes of motion of the manipulator links are important in forming the dependence of the change in the combined moment of inertia of the manipulator. For example, the nature of the change in the moments of inertia of the boom for the parabolic mode of motion, which determined by the next functions, it was investigate:

$$\alpha = \alpha_0 + \frac{t^3(t_1\omega_{1\_max} - 2\Delta\alpha)}{t_1^3} - \frac{t^2(t_1\omega_{1\_max} - 3\Delta\alpha)}{t_1^2}; \quad (31)$$

$$\beta = \beta_0 + \frac{t^3(t_1\omega_{2\_max} - 2\Delta\beta)}{t_1^3} - \frac{t^2(t_1\omega_{2\_max} - 3\Delta\beta)}{t_1^2}; \quad (32)$$

$$\dot{\alpha} = \frac{3t^2(t_1\omega_{1\_max} - 2\Delta\alpha)}{t_1^3} - \frac{2t(t_1\omega_{1\_max} - 3\Delta\alpha)}{t_1^2}; \quad (33)$$

$$\dot{\beta} = \frac{3t^2(t_1\omega_{2\_max} - 2\Delta\beta)}{t_1^3} - \frac{2t(t_1\omega_{2\_max} - 3\Delta\beta)}{t_1^2}, \quad (34)$$

where: t_1 acceleration time; $\Delta\alpha$ and $\Delta\beta$ coordinate increment ($\Delta\alpha = \alpha_к - \alpha_0$; $\Delta\beta = \beta_к - \beta_0$, later it was accepted α_0 = 0,5 rad.; β_0 = 0,4 rad.; $\alpha_к$ = 1 rad.; $\beta_к$ = 0,8 rad.).

This mode of movement is characterize by rapid acceleration of the manipulator links on a given distance of the path. The graphs of this mode of movement were shows in Fig. 3.

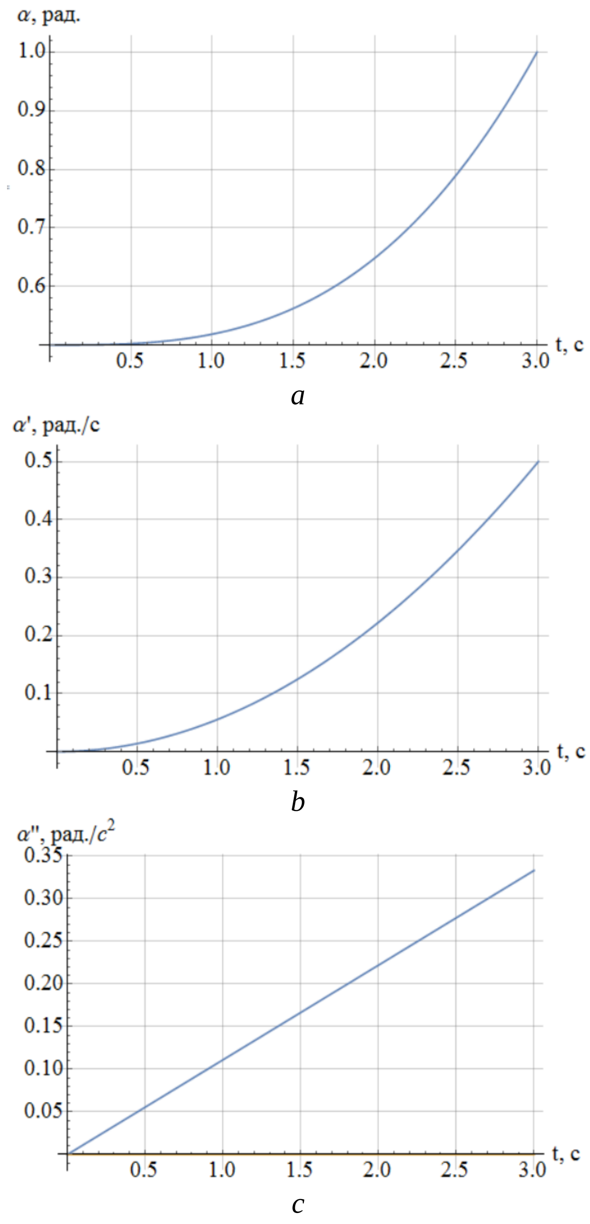

Fig. 3. Graphs of changes in the angle of rotation α (*a*) angular velocity dα/d*t* (*b*) and angular acceleration $d^2\alpha/dt^2$ (*c*)

The combined moment of inertia of the boom was also calculate for the given mode of movement. On the graphs from Fig. 4 shows the results of the study, which shows the change in the combined moments of inertia of the boom $J_{зв1}$, $J_{зв2}$.

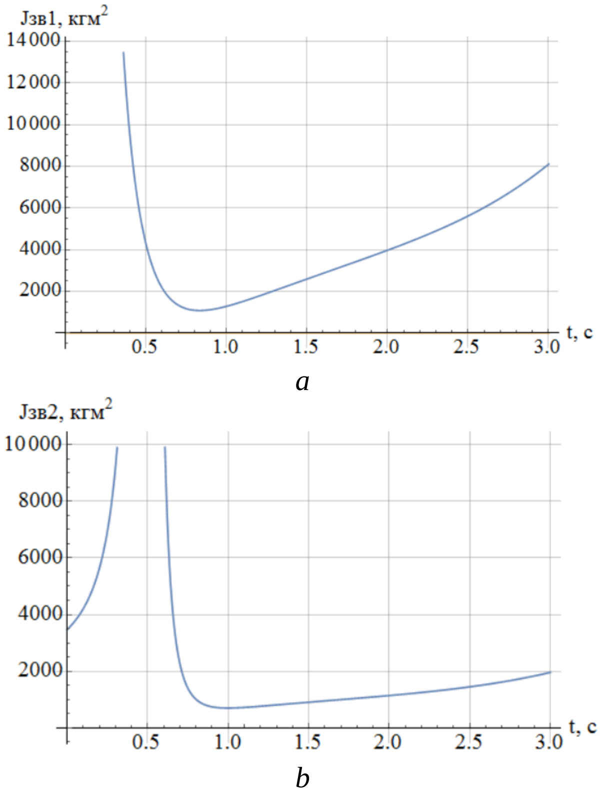

Fig. 4. Graphs of changes in combined moments of inertia $J_{зв1}$ (*a*) and $J_{зв2}$ (*b*)

From the graphs in Fig. 4 shows an increase in the combined moments of inertia of the boom during the movement of the manipulator links. However, at the initial stage of acceleration of the manipulator boom mechanism, there is a significant significant increase in the value of the combined moments of inertia, which gradually decreases rapidly, and after a time interval of 1 s, it begins to increase monotonically again, similarly to the previous considerations.

In Fig. 5 and Fig. 6 shows the results of the study of changes in static and dynamic moments in the drive mechanisms of the manipulator.

From the graphs in Fig. 2, it can be seen that the increase in the combined moments of inertia reaches $\Delta J_{зв1}$=7000 kg·m^2 and $\Delta J_{зв2}$= 1800 kg·m^2 for the motion mode with a linear speed change. The maximum values are 12000 kg·m^2 and 3000 kg·m^2 by $J_{зв1}$ and $J_{зв2}$. Fig. 4 shows that for the mode of movement with a parabolic change in speed, similar values are different. In particular, during the period of

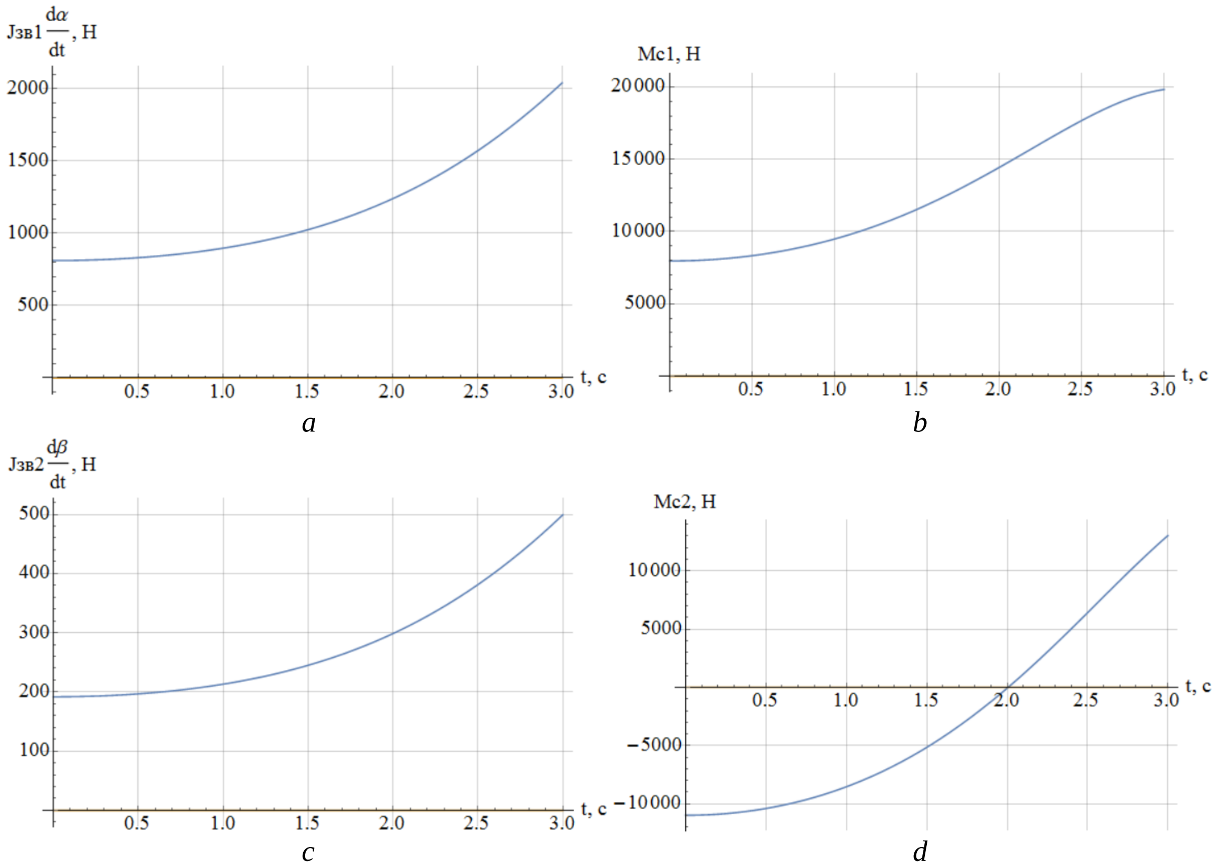

Fig. 5. The magnitude and nature of the change in dynamic (*a*, *c*) and static (*b*, *d*) moments of the drive under a typical driving mode with a linear characteristic of the change in angular velocity

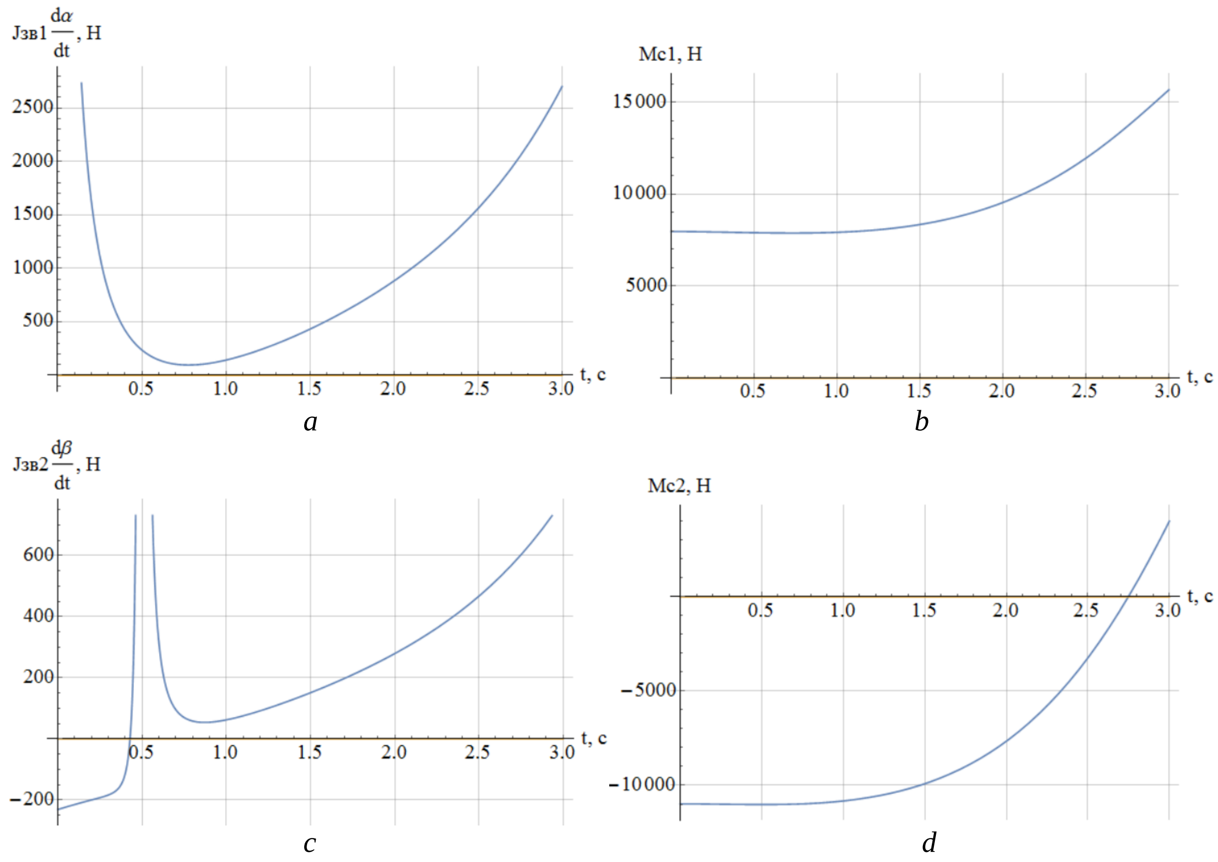

Fig. 6. The magnitude and nature of the change in dynamic (*a*, *c*) and static (*b*, *d*) moments of the drive in the dynamic mode of motion with a parabolic characteristic of the change in angular velocity

movement from 0 to 0.7 s the difference between the maximum and minimum values reaches almost infinity, and in the interval from 0.7 to 3 s the difference will be $\Delta J_{зв1}$ = 7400 kg·m^2 and $\Delta J_{зв2}$= 900 kg·m^2.

The static torques of the actuator were determinate by the next formulas:

$$M_{c1} = (0{,}5m_1 + m_2 + m_b)gl_1\cos\alpha + + (0{,}5m_2 + m_b)gl_2\cos\varphi; \quad (35)$$

$$M_{c2} = (0{,}5m_2 + m_b)gl_2\cos\varphi. \quad (36)$$

CONCLUSIONS

The conducted studies demonstrate how complex the dynamic process is in the boom system of the manipulator. The load on the drive elements depends on dynamic and static torque. Depending on the operating conditions of the manipulator, there are non-linear relationships between dynamic and static drive torques. The movement mode of the manipulator links significantly affects the change in the dynamic characteristics of the drive load. Thus, when choosing the required movement mode of the manipulator, it is necessary to apply optimization methods that allow taking into account the non-uniformity of changes in the dynamic parameters of the mechanical system.

REFERENCES

1. **Mishchuk, D., Boychenko, A.** (2019). Development of the concept of control system work for plaster works based on neural network. Girnichi, budivelni, dorozhni ta meliorativni mashini, Nr.93, 46–60. https://doi.org/10.32347/gbdmm2019.93.0501.
2. **Korobenko, Y., Mishchuk, D., Sankin, I.** (2023). Research of relationships between the technical parameters of industrial manipulators. Girnichi, budivelni, dorozhni ta meliora-

tivni mashini, Nr.102, 65–73. https://doi.org/ 10.32347/gbdmm.2023.102.0502.

3. **Mishuk D. O.** (2014). Kinematichnij analiz manipulyacijnoyi sistemi budivelnogo robota [Kinematic analysis of the manipulation system of a construction robot]. Vestnik Harkovskogo nacionalnogo avtomobilno-dorozhnogo universiteta, Vol. 65-66, 90-96. – (*in Ukrainian*).
4. **Zhang X. Y., Zhang T. X., Cai Y. Q.** (2024). Design and analysis of automatic opening and closing manipulator for ore crushing ball mill cover. Metalurgija, 63(3-4), 443-446.
5. **Loveikin V. S., Romasevych Yu. O., Spodoba O. O., Loveikin A. V., Pochka K. I.** (2020). Mathematical model of the dynamics change departure of the jib system manipulator with the simultaneous movement of its links. Strength of Materials and Theory of Structures, Nr.104, 175-190.
6. **Loveikin V., Mishchuk D.** (2022). Analysis of the refined mathematical model of the manipulator boom system with an elastic base. Girnychi, budivelni, dorozhni ta melioratyvni mashyny [Mining, construction, road and melioration machines], Nr.99, 5-14.
7. **Mora-Aquino, G., Rodríguez-Morales, A. L., López-Huerta, F., Elvira-Hernández, E. A., Herrera-May, A. L.** (2024). Recent advances in bioinspired walking microbots: Design, manufacturing, and challenges. Sensors and Actuators A: Physical, 372.
8. **Loveykin V., Mishchuk D.** (2023). Optimal mode movement of the robot manipulator on an elastic base according to the criterion of the mean square value of the acceleration of the drive torque. Girnichi, budivelni, dorozhni ta meliorativni mashini, Nr.101, 11–20.
9. **Duc D. N., Viet T. V.** (2024). Adaptive synchronous sliding control for a robot manipulator based on neural networks and fuzzy logic. International Journal of Electrical and Computer Engineering, 14(3), 2377-2385.
10. **Loveykin V. S., Mishchuk D. O., Mishchuk Ye. O.** (2022). Optimization of manipulator's motion mode on elastic base according to the criteria of the minimum central square value of drive torque. Strength of Materials and Theory of Structures, Issue 109. https://doi.org/ 10.32347/2410-2547.2022.109.403-415.
11. **Loveykin V. S., Mishchuk, D. O., Romasevych, Y. O.** (2023). The movement mode optimization of the manipulator on the elastic base according to the criterion of the mean square value of the rate of change of the drive torque. Opir materìalìv i teoria sporud [Strength of Materials and Theory of Structures]. https://doi.org/10.32347/2410-2547.2023.110.457-468.
12. **Voliyanuk V., Mishchuk D., Parkhomenko M.** (2022). Modeling dynamic control model of a two-link crane-manipulator. Girnichi, budivelni, dorozhni ta meliorativni mashini [Mining, construction, road and reclamation machines], Nr.99, 15–19. https://doi.org/ 10.32347/gbdmm.2022.99.0201
13. **Mischuk D. Volianiuk V., Gorbatyuk Eu.** (2019). Determination of geometric parameters of the manipulator according to the characteristics of the working environment. Girnichi, budivelni, dorozhni ta meliorativni mashini [Mining, construction, road and reclamation machines], Nr.94, 25-34. (*in Ukrainian*). https://doi.org/10.32347/gbdmm2019.94.0201
14. **Volianiuk V., Mischuk D., Gorbatyuk Eu.** (2022). Determination of dynamic loads in the crane lifting mechanism. Girnichi, budivelni, dorozhni ta meliorativni mashini [Mining, construction, road and reclamation machines], Nr.99, 15-25. (*in Ukrainian*).
15. **Volianiuk V., Mischuk D., Gorbatyuk Eu.** (2020). Determination of inertial loads of the rotary boom of a self-propelled crane. Girnichi, budivelni, dorozhni ta meliorativni mashini [Mining, construction, road and reclamation machines], Nr.96, 13-21. (*in Ukrainian*). https://doi.org/10.32347/gbdmm2020.96.0201
16. **Loveykin V. S., Mischuk D. O.** (2013). Optimization of the modes of changing the departure of the manipulator with a hydraulic drive. Monograph, Kyiv, CP Komprint, 206. https://doi.org/10.26884/damu.m13opzvmg
17. **Mishchuk D. O.** (2017). Study of the dynamics of the boom manipulator mounted on an elastic sup-port. Girnychi, budivelni, dorozhni ta melioratyvni mashyny [Mining, construction, road and melioration machines], Nr.90, 11-18.
18. **Onose R., Sawada H. A** (2024). Ball-jointed tendon-driven continuum robot with multidirectional operability for grasping objects. ROBOMECH Journal, 11(1), art. no. 4.
19. **Hedberg E.** (2020). Control, models and industrial manipulators. LiU-Tryck, Linköping, Sweden, 1-68.

Дослідження зміни зведеного моменту інерції дволанкового маніпулятора з кутовою системою координат

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Анотація. Одна із проблем розвитку дволанкових маніпуляторів з кутовою системою координат є відсутність ефективних і універсальних систем управління, які здатні реалізовувати енергоємні режими руху. Це пов'язано з особливостями конструкції стрілової системи таких маніпуляторів та способами їхнього застосуванням. В процесі роботи маніпулятора в його системі приводу виникають значні динамічні навантаження, які створюють коливання механічної системи та зменшують точність виконання робіт. Система керування також повинна ефективно компенсувати такі динамічні коливання.

Для побудови контролерів управління маніпуляторами застосовують динамічні рівняння руху, які дозволяють враховувати різні особливості використання маніпуляторів. Такі математичні моделі обов'язково містять одну із характеристик у вигляді зведеного моменту інерції механічної системи, яка є важливою при розрахунку диференційних рівнянь та визначення кінематичних параметрів руху.

В даній статті визначено зведені моменти інерції стрілової системи дволанкового маніпулятора з кутовою системою координат. Було проведено дослідження та показано, що на характер зміни зведених моментів інерції впливають режими руху ланок стрілової системи.

Результати дослідження показали, що зведений момент інерції стрілової системи маніпулятора є змінною величино, яка не рівномірно змінюється при особливих умовах роботи машини.

Для визначення зведених моментів інерції було застосовано методику з приведення кінетичної енергії рухомої системи до приводного механізму. Це є актуальним, бо саме приводом в подальшому потрібно буде здійснювати регулювання нерівномірності руху ланок маніпулятора.

Дослідження було проведено для типового лінійного режиму руху та режиму руху зі зміною швидкості за параболічною траєкторією.

Ключові слова: дволанковий маніпулятор, кінетична енергія, зведений момент інерції, динамічна модель, рівняння руху.