UDC 621.87

# Determination of dynamic loads in the crane suspension when lifting a load from a rigid base

*Volodymyr Volianiuk*[1], *Dmytro Mishchuk*[2], *Eugene Gorbatyuk*[3]

[1,2,3] Kyiv National University of Construction and Architecture,
31 Air Force Avenue, Kyiv, Ukraine, 03037,
[1]volian535@ukr.net, https://orcid.org/0000-0002-6852-9037,
[2]tdmid@ukr.net, https://orcid.org/0000-0002-8263-9400,
[3]gek_gor@i.ua, https://orcid.org/0000-0002-8148-5323

*Received: 25.01.2024; Accepted: 29.03.2024*
https://doi.org/10.32347/gbdmm.2024.103.0201

**Abstract.** Loading and unloading works are an integral part of the construction process. Cranes of various types were mostly use to perform these works.

To ensure trouble-free operation and increase the reliability of cranes, when calculating structures and components of their working equipment, it is important to take into account dynamic loads, which are several times higher than static loads. Elements of dynamic loads in the crane suspension are its elastic components (flexible traction bodies) - ropes.

The process of lifting a load from a rigid base and picking it up is considered, which divided into three stages: the first is the selection of clearances and the tension of the ropes; the second is the pre-opening stage of lifting the load; the third is the post-detachment stage of lifting the load.

For each stage, the initial conditions accepted, the differential equations of the movement of loads compiled, their solution given taking into account many factors, and expressions derived for determining the forces in the load suspension. At the first stage, the duration of the gap selection (tension of the ropes) is determined, at the second stage, the speed of separation of the load from the base is determined, at the third stage, the maximum force in the elastic element determined.

The method of determining the forces in the suspension of the load, the duration of the selection of clearances (tension of the ropes), the speed of separation of the load from the base, and the maximum force in the elastic element presented in the work allows you to significantly simplify the solution of complex equations, to determine simple expressions and to determine them with sufficient accuracy for practical calculations values.

**Keywords:** crane, mechanism, load, moment, effort.

## INTRODUCTION

Cranes, as lifting machines, are widely used in construction to move loads and install structures.

Scientific and technical progress, which is taking place in all countries of the world, constantly requires an increase in productivity, load capacity and an increase in the working speed of lifting machines. This leads to a reduction in transient processes, i.e. to a reduction in the acceleration and braking time of machines.

All this leads to an increase in the workload of the lifting machine, there are additional efforts on all the machine elements, which received the name in technology - external dynamic loads.

On the other hand, any machine has structural features of its kinematics. Deformation of flexible elements during the operation of the machine creates oscillating processes in the load suspension and refers to the phenomena of the internal dynamics of the machine.

For the safe operation of cranes, it is important to consider the amount of all types of dynamic loads when calculating their structures and selecting components.

## PURPOSE OF THE ARTICLE

Develop a methodology for determining dynamic loads in the crane suspension when lifting a load from a rigid base in order to simplify complex calculations and determine simple expressions for practical calculations.

## ANALYSIS OF PREVIOUS RESEARCH

In work [1] V. S. Loveykin, Yu. V. Chovniuk, M.D. Dikteruk and S.I. Pastushenko, the tasks of dynamic analysis and synthesis of movement modes of the load lifting mechanism were set and solved. This work uses systems with concentrated and distributed parameters (crane bridge, rope). The results of the research illustrated by graphic dependencies.

V. F. Gaidamaka studied the dynamic forces that arise in the load lifting mechanism under the condition that the speed of the rope hitting the drum in the pre-break-off stage of the load's movement is constant, and in the post-break-off stage the driving force is constant [2]. The cargo lifting mechanism presented in the form of a two-mass dynamic model. To model the loading mode, the author uses a piecewise-constant function of changing the braking force.

In article [3], an analysis of dynamic loads in non-linear elastic elements of the load lifting mechanism of bridge cranes at the stage of their start-up carried out. This is a generalization of the linear version of the model (crane lifting mechanism), where a two-mass dynamic model with a linear elastic rope is used.

In the articles [4, 5], the dependences for determining the inertial loads in the mechanisms of self-propelled cranes when lifting (lowering) the load, turning the boom with the load, raising (lowering) the boom and combinations of these movements are given.

Work [6] was state that the load lifting mechanism consists of a large number of elastic elements. Compiling and solving equations to determine the elasticity coefficients is a difficult task. In order to simplify the equations and these calculations, the next scheme of calculations was recommend in the work, according to which the rest of all the elements of the mechanism reduced to its first element (engine). This allows you to significantly simplify the equations for solving and determine the values of the elasticity or stiffness coefficients of the elements of the dynamic loads of the load lifting mechanism of cranes.

In work [7], the dynamic response of tower cranes in combination with pendulum movements of the payload investigated. To simplify the main equation, a simple scheme of disturbances and the assumption of a small pendulum angle are used. The integrated governing equations were derive on the Lagrange's equations.

The dynamic loads during the rotary movement of a tower crane with a load, which causes the spatial movement of the pendulum were consider in works [8, 9]. For this, nonlinear mathematical models of load oscillation during turning movement were formulate, taking into account the nonlinearity of the rocking movement at large angles and the nonlinearity of power transmission.

The bridge crane modeled as a point mass was consider in [10]. A payload of point mass is attached to the carriage by means of a massless beam and moves both in-plane and out-of-plane. The influence of pendulum movement, pendulum length and mass of payload on pendulum movement was studies.

The work [11] proposed a method of exact integration for calculating the dynamic loads of the system. This method has the advantages of high accuracy and takes less computing time, is simple and effective for analyzing the dynamic characteristics of the crane system.

All cited works and publications, the indicators of the dynamic load of the crane suspension are not determined.

## PRESENTING MAIN MATERIAL

Consider the dynamic loads in the lifting mechanism when the load is on a rigid base during its lifting.

When accelerating the lifting mechanism, when the load is on a rigid base, two cases are distinguished:

1. Starting the mechanism with elastic pickup of the load.
2. Start with picking up the load.

The start-up with elastic pick-up means the case when the acceleration of the mechanism begins when the load lowered onto the support with the tension of the ropes, i.e. $0 < F < G_{ван}$, where $F$ is the tension of the ropes, kH; $G_{ван}$ is cargo weight, kH.

Starting with picking up the load means the case when the acceleration of the mechanism begins with the load lowered onto the support and the ropes weakened i.e. $F < 0$.

When lifting a load with pickup, depending on the tension of the ropes, the type of engine and the launch system, two cases are considered:

1. ropes can start to stretch before the end of the engine acceleration, i.e. $M_д < M_н$;
2. ropes can begin to stretch after full acceleration of the engine when $M_д = M_н$, where $M_д$ is actual engine torque, Nm; $M_н$ is nominal engine torque, Nm.

Dynamic loads in the second case are much higher than in the first.

When considering the dynamic loads in the crane suspension during start-up with pick-up, we first assume that the load is on a rigid base and consider the metal construction of the crane as rigid.

The process of lifting "with pick-up" is considered as a step-by-step process:

- the first step is the selection of clearances and the tension of the ropes;
- the second step is the pre-tear stage, when the force in the elastic element increases to the value «$G_{ван}$» at a fixed mass «$m_2$»;
- the third step is the post-detachment stage, which begins with the movement of the mass "m2" detached from the support.

**The first stage.**

At the first stage, the gap is selected «Δ»in this system. During the first stage, mass «$m_1$» moves under the influence of a constant mean starting force «$P_1$», at the same time, the gap is selected «Δ» (Fig.1).

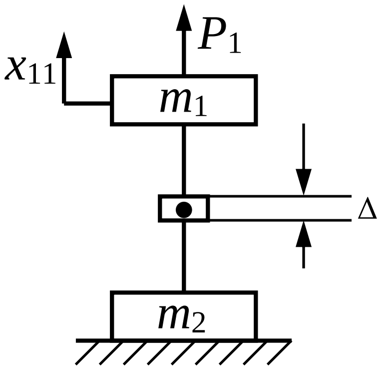


**Fig. 1.** Scheme of the first stage with cargo pickup

The equation of motion of the leading mass, at the first stage, was write down:

$$m_1 \ddot{x}_{11} = P_1 \quad (1)$$

where

$$\ddot{x}_{11} = \frac{P_1}{m_1}. \quad (2)$$

Integrating twice, it was get:

$$\dot{x}_{11} = \int_o^t \ddot{x}_{11} dt = \frac{P_1}{m_1} t; \quad (3)$$

$$x_{11} = \int_o^t \dot{x}_{11} dt = \frac{P_1}{2 \cdot m_1} t^2, \quad (4)$$

at x = Δ the duration of the first stage has been obtained:

$$t = \sqrt{\frac{2m_1 \Delta}{P_1}}, \quad (5)$$

where Δ is clearance or the amount of slack in the rope.

Accelerated pickup were considered in the case when the driving force depends linearly on the speed of the driven mass, which is characteristic of crane drives with an asynchronous electric motor or a direct current shunt motor. In the first approximation, it can be assumed that the electric motor works on one artificial characteristic at all stages of pick-up (Fig. 2).

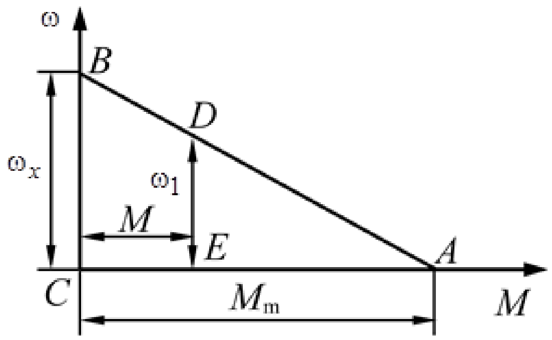


**Fig. 2.** Simplified artificial characteristics of the electric motor

On Fig. 2 was marked:
$M_m$ – maximum engine torque, Nm;
$M$ – is the current value of the engine torque, Nm;
$\omega_x$ – is the angular velocity of the engine idling, rad/s;
$\omega_1$ – the current value of the angular speed of the engine, rad./s.

From the similarity of triangles ABC and ADE it was write:

$$\frac{M_m - M}{\omega_1} = \frac{M_m}{\omega_x}; \ \frac{M_m}{\omega_1} - \frac{M}{\omega_1} = \frac{M_m}{\omega_x};$$

$$\frac{M_m}{\omega_1} - \frac{M_m}{\omega_x} = \frac{M}{\omega_1}, \qquad (6)$$

when it was multiply by the current velocity $\omega_1$, have been received:

$$M = M_m - M_m \frac{\omega_1}{\omega_x}; \ M = M_m - B\dot{x}_{11}, \quad (7)$$

where
$B = \frac{M_m i_з}{R_1 \omega_x}$ – the coefficient of proportionality;
$\omega_1 = \frac{\dot{x}_{11} i_з}{R_1}$ – the current angular velocity of the motor;
$i_з$ – drive reduction factor;
$R_1$ – drive drum radius.

Consider the case when the clearances are selected before the operator goes to the next stage of the artificial characteristic of the engine, then the equation of motion at the first stage of the mass movement «$m_1$» will be rewritten as follows:

$$m_1 \ddot{x}_{11} = \frac{i_з \eta_з}{R_1}(M_m - B\dot{x}_{11});$$

or

$$m_1 \ddot{x}_{11} + \frac{i_з \eta_з}{R_1} B\dot{x}_{11} = \frac{i_з \eta_з}{R_1} M_m. \qquad (8)$$

Let's divide the equation (8) by $m_1$

$$\ddot{x}_{11} + K_1 B\dot{x}_{11} = K_1 M_m, \qquad (9)$$

where $K_1 = \frac{i_з \eta_з}{m_1 R_1}$.

Integrate equation (9) by «$t$»

$$\dot{x}_{11} + K_1 B x_{11} = K_1 M_m t + A_{11}. \qquad (10)$$

Because at $t = 0$, $x_{11} = 0$, $\dot{x}_{11} = 0$, and therefore $A_{11} = 0$. Finally, will be receive:

$$\dot{x}_{11} + K_1 B x_{11} = K_1 M_m t. \qquad (11)$$

This is a differential equation of the first order, the characteristic equation of which:

$$\lambda_1 = -K_1 B = const.$$

In this case, the expression for mass displacement «$m_1$» will look like next:

$$x_{11} = A_{12} e^{\lambda_1 t} + A_{13} t + A_{14}, \qquad (12)$$

by $t = 0$ $x_{11} = 0$ and $\dot{x}_{11} = 0$, has been received:

$$0 = A_{12} + A_{14} \text{ or } A_{12} = -A_{14}.$$

Let us differentiate this equation and find the coefficients «$A_{12}$» and «$A_{14}$»:

$$\dot{x}_{11} = A_{12} \lambda_1 e^{\lambda_1 t} + A_{13}. \qquad (13)$$

Substituting into the first-order equation (11), grouping similar terms and transforming, the next equation it was obtain:

$$(A_{12}\lambda_1 e^{\lambda_1 t} + A_{13}) + \\ + K_1 B(A_{12} e^{\lambda_1 t} + A_{13} t + A_{14}) = K_1 M_m t;$$

or

$$A_{12}\lambda_1 e^{\lambda_1 t} + A_{13} + K_1 B A_{12} e^{\lambda_1 t} + K_1 B A_{13} t + \\ + K_1 B A_{14} = K_1 M_m t. \qquad (14)$$

It was group similar members by «$t$» and because $\lambda_1 = -K_1 B$ has been received next:



ISSN(online)2709-6149. Mining, constructional, road and melioration machines, 103, 2024, 15-24

$$-\lambda_1 A_{13} t + K_1 B A_{14} + A_{13} = K_1 M_m t\,. \quad (15)$$

By t = 0, from expression (15) it was have next:

$$A_{13} = \lambda_1 A_{14} \text{ або } A_{13} = -\lambda_1 A_{12}\,. \quad (16)$$

Substitute (16) into expression (15) and group the free terms:

$$-\lambda_1 A_{13} t - \lambda_1 A_{14} + \lambda_1 A_{14} = K_1 M_m t\,. \quad (17)$$

From expression (17), we have

$$A_{13} = -\frac{K_1 M_m}{\lambda_1} = -\frac{K_1 M_m}{-K_1 B} = \frac{M_m}{B} \quad (18)$$

or

$$A_{13} = \frac{M_m R_1 \omega_x}{M_m i_з} = \frac{R_1 \omega_x}{i_з}\,. \quad (19)$$

Then

$$A_{14} = \frac{A_{13}}{\lambda_1} = \frac{R_1 \omega_x}{-K_1 B i_з} = -\frac{m_1 R_1^3 \omega_x^2}{i_з^3 \eta_з M_m}\,. \quad (20)$$

Since $A_{12} = -A_{14}$ then $A_{12} = \frac{m_1 R_1^3 \omega_x^2}{i_з^3 \eta_з M_m}$.

Let's substitute the determined coefficients into the expression for moving the mass «$m_1$» at the first stage and we will get:

$$x_{11} = \frac{m_1 R_1^3 \omega_x^2}{i_з^3 \eta_з M_m}(e^{\lambda_1 t} - 1) + \frac{R_1 \omega_x}{i_з} t\,. \quad (21)$$

The first stage ends with a complete sampling of «Δ»gaps in the system, i.e $x_{11} = \Delta$.

Let's expand the exponent function into a series:

$$e^{\lambda_1 t} = 1 + \frac{\lambda_1 t}{1!} + \frac{(\lambda_1 t)^2}{2!} + \cdots + \frac{(\lambda_1 t)^n}{n!}\,, \quad (22)$$

where $\lambda_1 = -\frac{B}{m_1} = -\frac{M_m}{w_x m_1}$.

Let's leave the three terms of the decomposition from the equation and determine the duration of the first stage:

$$\Delta = \frac{m_1 R_1^3 \omega_x^2}{i_з^3 \eta_з M_m}\left(-\frac{i_з^2 \eta_з M_m}{m_1 R_1^2 \omega_x} t + \left(\frac{i_з^2 \eta_з M_m}{m_1 R_1^2 \omega_x}\right)^2 \frac{t^2}{2}\right) + \frac{R_1 \omega_x}{i_з} t = \frac{M_m i_з \eta_з}{2 m_1 R_1} t^2. \quad (23)$$

Whence, the duration of gap sampling (rope tension) at the first stage will be:

$$t = \sqrt{\frac{2\Delta m_1 R_1}{M_m i_з \eta_з}}\,. \quad (24)$$

System parameters at the border of the first and second stages:

$$\dot{x}_{11к} = \dot{x}_{12п} = -\frac{R_1 \omega_x}{i_з} e^{\lambda_1 t} + \frac{R_1 \omega_x}{i_з} = \frac{R_1 \omega_x}{i_з}(1 - e^{\lambda_1 t}); \quad (25)$$

$$M_{1к} = M_{2п} = M_m - \frac{M_m i_з}{R_1 \omega_x} \frac{R_1 \omega_x}{i_з} \times (1 - e^{\lambda_1 t}) = M_m e^{\lambda_1 t}. \quad (26)$$

**The second stage.**

The second stage is the pre-break-off stage, when the force in the elastic element increases to the value «$G_{ван}$» at the non-moving mass «$m_2$» (Fig. 3).

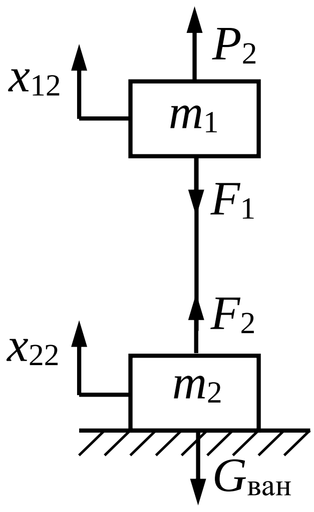


**Fig. 3.** Scheme of the second stage of cargo lifting

For the initial conditions at the second stage, we accept the final conditions of the first stage, i.e: mass «$m_1$» has an initial velocity $\dot{x}_{11} = V_x$, (м/с); displacement «$x_{11}$» and effort «$F_1$» are equal to zero.

At the second stage, the elastic link begins to be loaded to its maximum value «$G_{ван}$».

Differential equation of mass motion «$m_1$» will be write like this:

$$m_1 \ddot{x}_{12} = P_2 - F_2, \qquad (27)$$

since as $F_2 = G_{ван} + c\left(x_{12} - x_{22}\right)$, and at the same time $G_{ван} = 0$, $x_{22} = 0$, then this expression will be rewritten like this $F_2 = cx_{12}$, where $c$ – linear stiffness of cargo suspension.

Then, substituting the values $F_2$ we will get:

$$m_1 \ddot{x}_{12} + cx_{12} = P_2, \qquad (28)$$

or

$$\ddot{x}_{12} + \frac{c}{m_1} x_{12} = \frac{P_2}{m_1}. \qquad (29)$$

Since as $\omega^2 = c / m_1$, will finally be possessed:

$$\ddot{x}_{12} + \omega^2 x_{12} = \frac{P_2}{m_1}, \qquad (30)$$

where ω – natural frequency of free oscillations of the mass «$m_1$», s$^{-1}$, $P_2$ – driving force, kN.

If we substitute the value of the force "$F_2$" into the differential equation of motion (30), the driving force "$P_2$", which will be determined by the torque on the drive, then we will get the following equation:

$$\ddot{x}_{12} + \omega^2 x_{12} = \frac{i_з \eta_з}{m_1 R_1}(M_m - B\dot{x}_{11}),$$

or

$$\ddot{x}_{12} + K_1 B \dot{x}_{12} + \omega^2 x_{12} = K_1 M_m, \qquad (31)$$

where

$$K_1 = \frac{i_з \eta_з}{m_1 R_1}.$$

So how $\omega = \sqrt{\dfrac{c}{m_1}}$ – natural frequency of oscillation of the mass «$m_1$», c$^{-1}$, and $n = \dfrac{K_1 B}{2} = \dfrac{M_m i_з^2 \eta_з}{2R^2{}_1 \omega_x m_1}$ – coefficient characterizing the resistance of the medium.

By substituting, a differential equation of the second order is finally obtained, which is solved according to the rules for solving an equation of this form (the characteristic equation is compiled and its solution is found, then the solution is added taking into account the right-hand side):

$$\ddot{x}_{12} + 2n\dot{x}_{12} + \omega^2 x_{12} = K_1 M_m. \qquad (32)$$

The characteristic equation will be next:

$$\lambda^2 + 2n\lambda + \omega^2 = 0. \qquad (33)$$

If $n > \omega$, then the roots of the characteristic equation are distinct and real:

$$\lambda_1 = -n + \sqrt{n^2 - \omega^2}; \qquad (34)$$

$$\lambda_2 = -n - \sqrt{n^2 - \omega^2}. \qquad (35)$$

In this case, with real and unequal roots, the solution will be follow as:

$$x_{12} = A_{21} e^{\lambda_1 t} + A_{22} e^{\lambda_2 t} + x'_{12}, \qquad (37)$$

where $x'_{12}$ is partial inhomogeneous solution.

Let's determine the solution $x'_{12}$ in the form of a square polynomial:

$$x'_{12} = Ax_{12}^2 + Cx_{12} + D. \qquad (38)$$

Let's find the derivatives of expression (38) and substitute them into the basic differential equation (32):



ISSN(online)2709-6149. Mining, constructional, road and melioration machines, 103, 2024, 15-24

$$2A + 4nAx_{12} + 2nC + \omega^2 Ax_{12}^2 + \omega^2 Cx_{12} + \omega^2 D = K_1 M_m . \quad (39)$$

Equating the coefficients with the same powers $x_{12}$, it was get that A = C = 0 and

$$\omega^2 D = K_1 M_m \text{ whereof } D = \frac{K_1 M_m}{\omega^2} . \quad (40)$$

Since the beginning of the countdown of the second stage starts from the end of the first, when the velocity of mass movement «$m_1$» reached a value equal to $\dot{x}_{12п} = \frac{R_1 \omega_x}{i_з}(1 - e^{\lambda_1 t})$, and at the same time we will assume that $x_{12п}$ = = 0 and $t$ = 0. Equation (37) will be write follow as:

$$0 = A_{21} + A_{22} + \frac{K_1 M_m}{\omega^2},$$

where:

$$A_{21} = -A_{22} - \frac{K_1 M_m}{\omega^2} . \quad (41)$$

Then in the next equation

$$\dot{x}_{12} = A_{21}\lambda_1 + A_{22}\lambda_2 , \quad (42)$$

let's substitute the value «$A_{21}$»

$$\dot{x}_{12} = -A_{22}\lambda_1 - \frac{K_1 M_m}{\omega^2}\lambda_1 + A_{22}\lambda_2 , \quad (43)$$

whereof:

$$A_{22} = \frac{\dot{x}_{12}\omega^2 + K_1 M_m \lambda_1}{\omega^2(\lambda_2 - \lambda_1)} , \quad (44)$$

Substitute the value obtained «$A_{22}$» into equation (41) and find the first coefficient «$A_{21}$»:

$$A_{21} = -\frac{\dot{x}_{12}\omega^2 + K_1 M_m \lambda_1}{\omega^2(\lambda_2 - \lambda_1)} - \frac{K_1 M_m}{\omega^2} = = \frac{\dot{x}_{12}\omega^2 + K_1 M_m \lambda_2}{\omega^2(\lambda_1 - \lambda_2)} . \quad (45)$$

After that, when value «$x_{12}$» reaches such a value that the force in the elastic element becomes equal to the weight of the load «$G_{ван}$», the second stage ends, that is $F_{3п} = G_{ван} = cx_{12к.}$

We substitute the values in equation (37) $x_{12п} = G_{ван}/c$ and we will get the equation for determining the displacement when the load is separated from the base with duration «$t_2$»:

$$x_{12} = \frac{\dot{x}_{12}\omega^2 + K_1 M_m \lambda_2}{\omega^2(\lambda_1 - \lambda_2)} e^{\lambda_1 t} + \frac{\dot{x}_{12}\omega^2 + K_1 M_m \lambda_1}{\omega^2(\lambda_2 - \lambda_1)} e^{\lambda_2 t} + \frac{K_1 M_m}{\omega^2} = \frac{G_{ван}}{c} . \quad (46)$$

Then we differentiate the equation (46) by time and substitute values there «$t_2$» and we find the speed of the first mass at the moment of separation of the load from the base:

$$V_{відр} = \dot{x}_{12} = \dot{x}_{13п} = = \frac{\dot{x}_{12}\omega^2 + K_1 M_m \lambda_2}{\omega^2(\lambda_1 - \lambda_2)} e^{\lambda_1 t_2} + \frac{\dot{x}_{12}\omega^2 + K_1 M_m \lambda_1}{\omega^2(\lambda_2 - \lambda_1)} e^{\lambda_2 t_2} . \quad (47)$$

The main force, kN:

$$F_{3п} = cx_{12к} ; \quad \dot{F}_{2\,к} = cV_{відр} . \quad (48)$$

**The third stage.**

The third stage is the post-detachment stage, which begins with the movement of the mass «m₂» detached from the support.

At the third stage, both masses in the system will move (Fig. 4).

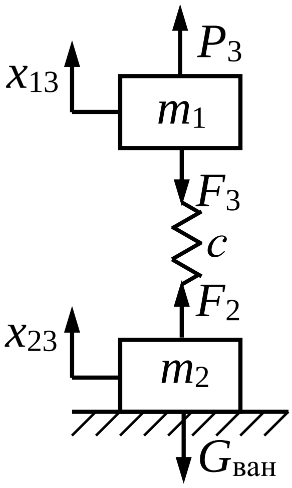


**Fig. 4.** Scheme of the third stage

The differential equation of motion of both masses will be write follows as:

$$\begin{cases} m_1 \ddot{x}_{13} = P_3 - F_3 \\ m_2 \ddot{x}_{23} = F_3 - G_{ван} \end{cases} \quad (49)$$

In this period, the driving force «$P_3$» differs little from the weight of the cargo, therefore, in the first approximation, it can be assumed that $P_3 = G_{ван}$.

From the differential equation of motion, we find the values of the accelerations of both masses:

$$\ddot{x}_{13} = \frac{G_{ван} - F_3}{m_1}; \quad (50)$$

$$\ddot{x}_{23} = \frac{F_3 - G_{ван}}{m_2}. \quad (51)$$

Differentiate the elastic load equation twice and substitute the mass acceleration value:

$$\ddot{F}_3 = \left(\ddot{x}_{13} - \ddot{x}_{23}\right)c = c\left(\frac{G_{ван} - F_3}{m_1} - \frac{F_3 - G_{ван}}{m_2}\right) =$$

$$= \frac{cG_{ван}}{m_1} - \frac{cF_3}{m_1} - \frac{cF_3}{m_2} + \frac{cG_{ван}}{m_2} =$$

$$= cG_{ван}\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - cF_3\left(\frac{1}{m_1} + \frac{1}{m_2}\right);$$

$$\ddot{F}_3 + cF_3\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = cG_{ван}\left(\frac{1}{m_1} + \frac{1}{m_2}\right);$$

$$\ddot{F}_3 = \omega^2 F_3 = G_{ван}\omega^2. \quad (52)$$

A linear inhomogeneous differential equation of the second order with constant coefficients was obtain. Where $\omega = \sqrt{c\frac{m_1 + m_2}{m_1 m_2}}$ - the frequency of self-oscillations of masses in the post-separation period.

The general solution of this equation will be write down:

$$F_3 = A\cos\omega t + B\sin\omega t + F_3'; \quad (53)$$

at the same time we will accept:

$$F_3' = D = const \quad \text{as well as} \quad \dot{F}_3 = \ddot{F}_3 = 0.$$

By substituting, these values in the differential equation (52) received next:

$$D\omega^2 = G_{ван}\omega^2; \quad (54)$$

$$D = G_{ван} \quad \text{i} \quad F_3' = G_{ван}.$$

Substituting (53) into the general solution of the differential equation, we obtain:

$$F_3 = A\cos\omega t + B\sin\omega t + G_{ван}. \quad (55)$$

Initial conditions of the third stage:

$$\begin{aligned} &F_{2к} = G_{ван} = cx_{12к}; \\ &\dot{F}_{2к} = c\dot{x}_{12к} = \dot{F}_{3п} = c\dot{x}_{13п} = cV_{відр}; \\ &F_{3п} = G_{ван}, \end{aligned} \quad (56)$$

Then at $t = 0$ the differential equation will be written:

$$G_{ван} = A + G_{ван} \text{ or } A = 0. \quad (57)$$

To find the second coefficient, we write down the first derivative equation (55):

$$\dot{F}_3 = -A\omega \sin \omega t + B\omega \cos \omega t, \qquad (58)$$

whereas $\dot{F}_3 = cV_{\text{відр}}$ then $cV_{\text{відр}} = B\omega$ whereof $B = \frac{cV_{\text{відр}}}{\omega}$.

Substitute and finally obtain the forces in the elastic element:

$$F_3 = \frac{cV_{\text{відр}}}{\omega} \sin \omega t + G_{\text{ван}}. \qquad (59)$$

It follows that the force in the elastic element after the load is detached $m_2$ from the support fluctuates around the value «$G_{\text{ван}}$» with amplitude «$\frac{cV_{\text{відр}}}{\omega}$» and circular frequency «ω» (Fig. 5).

The maximum value of the force on the rope in the break-off period will be at $t = \pi/2\omega$, then:

$$F_{\max} = G_{\text{ван}} + \frac{cV_{\text{відр}}}{\omega},$$

or

$$F_{\max} = G_{\text{ван}} + V_{\text{відр}} \sqrt{\frac{c(m_1 m_2)}{m_1 + m_2}}. \qquad (60)$$

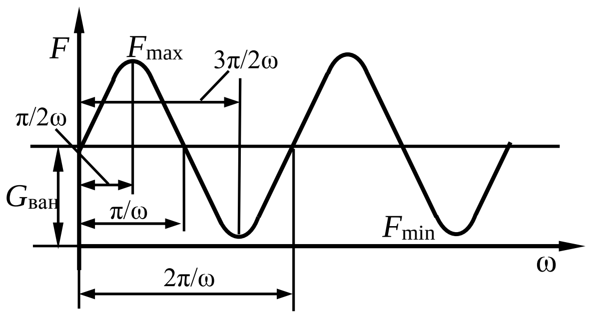


**Fig. 5.** The graph of the fluctuation of the force in the elastic element after the separation of the load from the base

It follows from this expression that the dynamic maximum force in an elastic element is directly proportional to its stiffness «c» and the speed of the load at the time of separation from the base.

In some cranes, the kinetic energy of the rotating mechanism masses is many times greater than the kinetic energy of the nominal lifting load, i.e. $m_1 >> m_2$, especially in general purpose cranes in which $m_1 = (10\ldots20)m_2$. For such mechanisms, it can be assumed that $m_1 + m_2 \cong m_1$, and when picking up the load from the support, the speed of mass movement does not change, i.e $\dot{x}_{11к} = \dot{x}_{12п} = \dot{x}_{12к} = \dot{x}_{13п} = V_{\text{відр}}$.

Then, substituting these values into the previous equation (60), we get the maximum force in the elastic element:

$$F_{\max} = G_{\text{ван}} + V_{\text{відр}} \sqrt{cm_2}. \qquad (61)$$

In fact, the value of the maximum force is slightly less than the value obtained by this equation. However, the simplicity and clarity of the physical meaning of this expression allow it to be used in preliminary practical calculations.

## CONCLUSIONS

The application of the given technique allows to determinate simple expressions for the calculations of dynamic load in the crane suspension when lifting the load from a rigid base with sufficient accuracy for practice. This simplifies calculations and reduces their duration.

In the future, it is necessary to develop programs to perform these calculations using computers.

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## Визначення динамічних навантажень в підвісці крана при підійманні вантажу з жорсткої основи

*Володимир Волянюк*[1], *Дмитро Міщук*[2], *Євгеній Горбатюк*[3]

[1,2,3]*Київський національний університет будівництва і архітектури*

**Анотація.** Навантажувально-розвантажувальні роботи є невід'ємною складовою технологічного процесу будівництва. Для виконання цих робіт здебільшого застосовують крани різних типів.

Для забезпечення безаварійної роботи і підвищення надійності кранів при розрахунках конструкцій і комплектуючих елементів їх робочого обладнання важливо враховувати динамічні навантаження, які в декілька разів перевищують статичні навантаження. Елементами динамічних навантажень в підвісці крана є його пружні складові (гнучкі тягові органи) – канати.

Розглянуто процес підіймання вантажу з жорсткої основи з його підхопленням, який розділяється на три етапи: перший - вибір зазорів і натяг канатів; другий – довідривна стадія підіймання вантажу; третій – післявідривна стадія підіймання вантажу.

Для кожного етапу прийняті початкові умови, складені диференціальні рівняння руху вантажів, наведено їх розв'язок з урахуванням багатьох чинників і виведені вирази для визначення зусиль в підвісці вантажу, тривалості вибору зазорів (натягу канатів) на першому етапі, швидкості відриву вантажу від основи на другому етапі, максимального зусилля в пружному елементі на третьому етапі.

Наведена в роботі методика  визначення зусиль в підвісці вантажу, тривалості вибору зазорів (натягу канатів) на першому етапі, швидкості відриву вантажу від основи на другому етапі, максимального зусилля в пружному елементі на третьому етапі дозволяє значно спростити розв'язання складних рівнянь, визначити прості вирази і з достатньою для практичних розрахунків точністю визначати їх величини.

**Ключові слова:** кран, механізм, навантаження, момент, зусилля.