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Structural modeling of a damper system using the method of bond graphs

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Abstract. The construction and analysis of the equations of the control system in the space of the state vector x is called the state space method. The modern theory of vibration protection systems is based on the concept of state space. Vibration protection systems use not only mechanical, but also other electronic, thermal, and hydraulic methods of energy conversion: such conversions are almost always carried out on one object. Due to these circumstances, the concept of state requires the expansion and use of such a set of variable states that its elements are universally adapted for use in various cases. Thus, this approach is based on topological representations of the continuity of energy change and conversion. This concept is the most modern and is the basis of a special modeling language using connection graph methods. The role of adaptive dampers, which can change their characteristics, is significant in modern mechanical engineering and construction. In turn, the graph model allows you to quickly change the structure of dampers and study the influence of parameters on the dynamics of the system. The method of the language of communication graphs allows obtaining matrix equations of vibration protection systems in the state space, in the form of state and observation equations. This approach makes it possible to consider many different types of vibration protection systems with a dynamic damper, including in a nonlinear formulation, as well as with controlled characteristics of the fluid element. The paper considers the modeling of vibration protection systems that have a dynamic vibration damper. The main assumptions necessary for the construction of a vibration protection system with a dynamic vibration damper are formulated. Nonlinear inertia and resistance functions are derived, which will provide the possibility of modeling the described features of the internal motion of the fluid in . A communication graph is constructed for the nonlinear formulation of the problem. The state equation and observation equation are compiled.

Keywords: communication graphs, dynamic oscillation damper, optimization, observation equation, equation of state.

INTRODUCTION

The construction and analysis of the control system equations in the space of the state vector x is called the state space method [1]. The modern theory of vibration protection systems is based on the concept of state space [2]. The state space is a set X consisting of elements x(t), which are defined by the vector [x_i], and =1, 2 ,..., n for a particular system [2]. For dynamic systems of mechanics, the state is defined by a pair of variables: the coordinate x and the velocity v or the displacement x, and the momentum q, which form the so-called state space or phase space [3],[4].

In vibration protection systems, not only mechanical, but also other electronic, thermal, and hydraulic methods of energy conversion are used: such conversions are almost always carried out on one object [5]. Due to these circumstances, the concept of a state requires the expansion and use of such a set of variable states that its elements are universally adapted for use in various cases [6]. From the point of view of energy, it is convenient to use energy dynamic variables for a universal description: potential e (t) and flow f (t), which determine the concentration and movement of energy, which is transformed in various ways during the functioning of the vibration protection system, while maintaining its continuity. Thus, this approach is based on topological representations of the continuity of energy change and conversion. This concept is the most modern and is the basis of a special modeling language using connection graph methods [7]. Using the language of connection graphs, it is possible to build a model of the vibration protection system in the form of a system of differential equations.

The role of adaptive dampers, which can change their characteristics, is significant in modern mechanical engineering and construction [8], [17]. In turn, the graph model allows you to quickly change the structure of dampers and study the influence of parameters on the dynamics of the system.

LITERATURE REVIEW

One of the methods for modeling technical systems is the method of functionally finite elements [9],[10]. When using this method, individual elementary material elements are distinguished in the system, which are considered as carriers of certain physical properties [11]. In addition, the system includes models of energy sources and converters of energy flow parameters [12]. The disadvantage of this method is the complexity of modeling nonlinear systems and systems in which different physical levels interact. In addition, there are difficulties in integrating the method into modern software complexes.

One of the directions of development of the theory of connection graphs is represented by a method that generates connection graphs from the Lagrange function [13], [14]. It produces correct but complex connection structures using complex formulas and inertial terms for transformer and gyrator modules [15]. In addition, it can be used with a discrete finite element scheme to simulate a wide range of problems of the dynamics of a solid continuous medium [16]. The disadvantages include poor adaptation of this method to modeling systems with strong damping and frictional losses.

In [18], a dynamic vibration damper is studied, represented as an elastic cantilever beam with a system of concentrated masses. The oscillations of the system are described by a boundary value problem with a discrete right-hand side. The disadvantage of this method of modeling systems is the lack of an explicit representation of the topology or structural connections between elements, and adaptability problems (changing one element of the system requires rewriting the entire boundary value problem).

GOAL AND PROBLEM STATEMENT

Modeling of a vibration protection system that has a dynamic vibration damper containing a special type of fluid element mounted on a supporting body. Building a model of a vibration protection system using the method of the language of connection graphs in linear and nonlinear problem formulation.

MAIN PART

A dynamic damper has a sealed chamber of various shapes, which is filled with a viscous incompressible fluid. Inside the chamber, a special type of solid body is placed on an elastic suspension, which is capable of performing translational oscillations along the chamber. These oscillations are transmitted to the fluid, the elastic suspension, the supporting body and provide vibration protection.

The method of the language of connection graphs allows to obtain matrix equations of vibration protection systems in the state space, in the form of equations of state and observation. This approach makes it possible to consider many different types of vibration protection systems with a dynamic damper, including in a nonlinear formulation, as well as with controlled characteristics of the fluid element.

When a dynamic damper moves in a liquid, it experiences resistance, which is caused by the viscosity of the liquid R_2 and its inertia Ia. In most cases, if we do not take into account the possibility of separation of I2 from the liquid, that is, cavitation is absent not only on the walls of the chamber, but also on the body of the dynamic damper itself, then we can sum up the inert properties of the solid body of the dynamic damper and the liquid I_2 + Ia = I_2a. The above assumptions provide a linear formulation of the

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problem, assuming them, we can build the simplest model of the vibration protection system, Fig. 1.

The following basic notations are adopted for the model: $I_1 : m_1$ - inertia (mass) of the supporting body; $I_2 : m_2$ - inertia (mass) of a solid body; $I_3 : m_3$ - inertia (mass) of the dynamic damper fluid; $Ia : m_a$ - inertia (mass involved) of the liquid; $R_1 : k_1$ - resistance (viscosity) of the suspension of the supporting body; R_2 ; k_2 - resistance (viscosity) of the dynamic damper suspension; C_I: c_I - elasticity (compliance) of the suspension of the supporting body; $C_3 : c_3$ - elasticity (compliance) of the dynamic damper suspension.

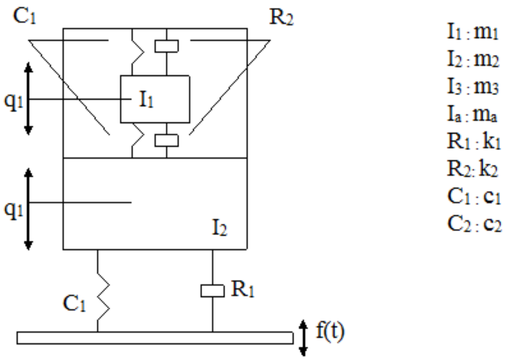

Fig. 1. Vibration protection object with vibration damper

Let us construct a communication graph that corresponds to the linear formulation of the problem. It should be noted that such a formulation of the problem is greatly simplified and does not take into account the complex dynamics of the internal motion of the fluid and the influence of this motion on the vibration protection effect. To overcome these difficulties, within the framework of the method of the language of communication graphs, nonlinear inertia and resistance functions can be introduced, which will provide the possibility of modeling the described features of the internal motion of the fluid in a dynamic damper. These considerations are reflected in the communication graph for the nonlinear formulation of the problem in Fig. 2:

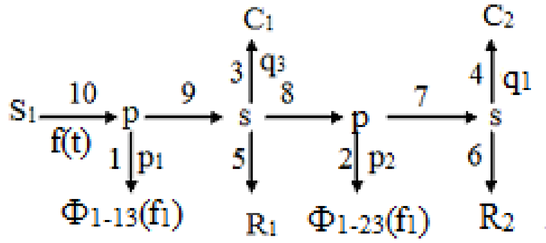

Fig. 2. Communication graph for nonlinear problem formulation

To model the nonlinear characteristics of the fluid inertia and its resistance, nonlinear functions are introduced: $\Phi_{I\,13}(f_1)$ - simulate the interaction of the chamber, fluid, solid body of the dynamic damper; $\Phi_{I\,23}(f_1)$ - model the nonlinearities that arise during the motion of a rigid body, a dynamic damper, and a fluid.

Let us compose, based on the connection graph, the equation of state. Let us denote by numbers the components of the connection graphs: momentum on inertia p_1 , p_2 and displacement on elasticity q_3, q_4; input vector of the stream variable source f_{10}. The state of the system is determined by the vector: (p_1, p_2, q_3, q_4).

The equations of state of the vibration protection system with dynamic damper in general form in the state space have the form:

$$p_1 = e_1;\ p_2 = e_2;\ q_3 = f_3;\ q_4 = f_4 \qquad (1)$$

Using the connection graph and the relation of the transitional node structure, we construct the defining equation for the transitional structures p, s:

$$\begin{aligned} &p:\left|\begin{array}{l} e_9 = e_1 = e_{10} \\ f_{10} = f_1 + f_9 \end{array}\right.;\quad s:\left|\begin{array}{l} f_9 = f_3 = f_5 = f_8 \\ e_0 = e_3 + e_5 + e_9 \end{array}\right.; \\ &p:\left|\begin{array}{l} e_3 = e_2 = e_7 \\ f_8 = f_2 + f_7 \end{array}\right.;\quad s:\left|\begin{array}{l} f_7 = f_4 = f_6 \\ e_7 = e_4 + e_6 \end{array}\right.. \end{aligned} \qquad (2)$$

Applying the relations for transition structures and the expressions for the elements I, R, C of the connection graphs, substituting them into the equation of states (for a linear problem) after transformations, we obtain:

$$\begin{aligned}
&\dot{p}_1 = -(R_1 + R_2) I_{13}^{-1} P_1 - R_2 I_{20}^{-1} p_2 + C_1^{-1} q_6 + \\
&+C_2^{-1} q_4 + \left(R_1 + R_2\right) f_{10}; \\
&\dot{p}_2 = -R_2 I_{13}^{-1} p_1 - R_2 I_{20}^{-1} p_2 + C_2^{-1} q_4 + R_2 f_{10}; \quad (3)\\
&\dot{q}_3 = -I_{13}^{-1} p_1 + f_{10}; \\
&\dot{q}_4 = -I_{13}^{-1} C_1 - I_{20}^{-1} C_2 + f_{10}.
\end{aligned}$$

For the nonlinear equation of state problem of vibration protection systems with a dynamic damper:

$$\begin{aligned}
&\dot{p}_1 = -(R_1 + R_2) \Phi_{I_{13}}^{-1}(p_1) - R_2 \Phi_{I_{20}}^{-1}\left(p_2\right) + \\
&+C_1^{-1} q_3 + C_2^{-1} q_4 + (R_1 + R_2) f_{10}; \\
&\dot{p}_2 = -R_2 \Phi_{I_{13}}^{-1}(p_1) - R_2 \Phi_{I_{20}}^{-1}(p_2) + \\
&+C_2^{-1} q_4 + R_2 f_{10}; \\
&\dot{q}_3 = -\Phi_{I_{13}}^{-1}(p_1) + f_{10}; \\
&\dot{q}_4 = -\Phi_{I_{13}}^{-1}(p_1) - \Phi_{I_{20}}^{-1}(p_2) + f_{10}.
\end{aligned} \tag{4}$$

The system of equations in matrix form has the following form:

$$\begin{vmatrix} \dot{p}_1 \\ \dot{p}_2 \end{vmatrix} = \begin{vmatrix} -\left(R_1 + R_2\right) I_{13}^{-1} & -R_2 I_{20}^{-1} & C_1^{-1} & C_1^{-1} \\ -R_2 I_{13}^{-1} p_1 & -R_2 I_{20}^{-1} & 0 & C_2^{-1} \end{vmatrix} \begin{vmatrix} p_1 \\ p_2 \end{vmatrix} + \\ + \begin{vmatrix} R_1 + R_2 \\ R_2 \end{vmatrix} \tag{5}$$

$$\begin{vmatrix} \dot{q}_3 \\ \dot{q}_4 \end{vmatrix} = \begin{vmatrix} -I_{13}^{-1} & 0 & 0 & 0 \\ -I_{13}^{-1} & -I_{20}^{-1} & 0 & 0 \end{vmatrix} \begin{vmatrix} q_3 \\ q_4 \end{vmatrix} + \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

Thus, based on the model of a vibration protection system with a dynamic damper, represented in the form of connection graphs, a system of differential equations in the state space of the form x\` = Ax + Bu where:

$$x = \begin{vmatrix} \dot{p}_1 \\ \dot{p}_2 \end{vmatrix} \quad A = \begin{vmatrix} -\left(R_1 + R_2\right) I_{13}^{-1} & -R_2 I_{20}^{-1} & C_1^{-1} & C_2^{-1} \\ -R_2 I_{13}^{-1} p_1 & -R_2 I_{20}^{-1} & 0 & C_2^{-1} \end{vmatrix}$$
$$x = \begin{vmatrix} \dot{q}_3 \\ \dot{q}_4 \end{vmatrix} \quad A = \begin{vmatrix} -I_{13}^{-1} & 0 & 0 & 0 \\ -I_{13}^{-1} & -I_{20}^{-1} & 0 & 0 \end{vmatrix} \tag{6}$$

The system of equations in matrix form is represented as:

$$\begin{aligned}
&\dot{p}_1 = -\left(R_1 + R_2\right) \Phi_{I_{13}}^{-1}\left(p_1\right) - R_2 \Phi_{I_{20}}^{-1}\left(p_2\right) + C_1^{-1} q_3 + \\
&+C_2^{-1} q_4 + \left(R_1 + R_2\right) f_{10}; \\
&\dot{p}_2 = -R_2 \Phi_{I_{13}}^{-1}\left(p_1\right) - R_2 \Phi_{I_{20}}^{-1}\left(p_2\right) + C_2^{-1} q_4 + R_2 f_{10}; \\
&\dot{q}_3 = -\Phi_{I_{13}}^{-1}\left(p_1\right) + f_{10}; \\
&\dot{q}_4 = -\Phi_{I_{13}}^{-1}\left(p_1\right) - \Phi_{I_{20}}^{-1}\left(p_2\right) + f_{10}.
\end{aligned} \tag{7}$$

The system of equations can be written in the following form: *x\` = F(x) + Bu* where:

$$\begin{aligned}
&\dot{p}_1 = -\left(R_1 + R_2\right) I_{13}^{-1} p_1 - R_2 I_{20}^{-1} p_2 + C_1^{-1} q_3 + \\
&+C_2^{-1} q_4 + \left(R_1 + R_2\right) f_{10}; \\
&\dot{p}_2 = -R_2 I_{13}^{-1} p_1 - R_2 I_{20}^{-1} p_2 + C_2^{-1} q_4 + R_2 f_{10}; \\
&\dot{q}_3 = -I_{13}^{-1} p_1 + f_{10}; \\
&\dot{q}_4 = -I_{13}^{-1} C_1 - I_{20}^{-1} C_2 + f_{10}.
\end{aligned} \tag{8}$$

In addition to the equations of state of the system under study, for its complete description, observation equations are required, which relate the state to the observable output parameter of the system. To find these equations, it is necessary to specify which parameter is the output of the system. In the case under consideration, this may be the displacement of mass I_{13} from the equilibrium superposition q1 or the velocity f4. Assuming that the state vector x is known, we obtain the observation equation:

If $y=q_1$, then for a linear problem

$$q_1 = \left[-\int_{13}^{-1} dt \right] x$$

For a nonlinear problem

$$q_1 = \left[-\int_{13}^{-1} \Phi dt \right] x$$

CONCLUSIONS

The obtained equations of vibration protection systems with a dynamic damper describe its behavior in the state space. The main method for solving these problems is to solve a system of linear and nonlinear first-order differential equations given in normal forms. Currently, various application software packages have been developed that provide the ability to quickly analyze the qualitative and quantitative behavior of the system, obtain its amplitude-frequency characteristics and optimize parameters.

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Структурне моделювання демпферної системи методом графів зв’язку

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Анотація. Побудова та аналіз систем управління зручно здійснювати в просторі вектора стану х тому що сучасна теорія віброзахисних систем базується на понятті простору стану. У системах віброзахисту використовуються не тільки механічні, а й інші електронні, термічні, гідравлічні методи перетворення енергії: майже завжди такі перетворення здійснюються на одному об’єкті. Завдяки цим обставинам поняття стану вимагає розширення й уживання такої множини змінні стани, щоб його елементи були універсально пристосовані до вживання в різноманітних випадках. Таким чином, в основу цього підходу покладені топологічні подання про безперервність зміни й перетворення енергії. Ця концепція найбільш сучасна й покладена в основу спеціальної мови моделювання методами графів зв'язку. Роль адаптивних демпферів, які можуть змінювати свої характеристики є значною в сучасному машинобудуванні та будівництві. В свою чергу графова модель дозволяє швидко змінювати структуру демпфер та досліджувати вплив параметрів на динаміку системи. Метод мови графів зв’язку дозволяє отримати матричні рівняння систем віброзахисту в просторі станів, у вигляді рівнянь стану і спостереження. Такий підхід дає можливість розглядати безліч різних видів віброзахисних систем з динамічним демпфером, у тому числі в нелінійній постановці, а також з керованими характеристиками рідинного елемента. В роботі розглянуто моделювання системи віброзахисту, які мають динамічний гаситель коливань. Сформульовано основні допущення необхідні для побудови віброзахисної системи з динамічним гасителем коливань. Виведені нелінійні функції інертності і опору, які забезпечать можливість моделювання описаних особливостей внутрішнього руху рідини в . Побудовано граф зв'язку для нелінійної постановки задачі. Складено рівняння стану та рівняння спостереження.

Ключові слова: графи зв’язку, динамічний гаситель коливань, оптимізація, рівняння спостереження, рівняння стану.