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# Determination of rheological parameters for tire–road contact system

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**Abstract.** The efficiency and safety of modern wheeled vehicles significantly depend on the reliable interaction between pneumatic tires and the supporting road or soil surface under varying load conditions. This interaction is governed by a complex combination of elastic, viscous and plastic deformation processes, occurring both in the tire structure and in the contact layer of the supporting surface. Accurate description of these processes requires the development and application of advanced rheological models capable of reproducing the nonlinear and time-dependent behavior of tire materials.

The analytical and experimental methodologies for determining the rheological parameters of pneumatic tires, including stiffness, deformation modulus, viscous resistance and internal friction characteristics are proposed in the paper. The methods are based on the analysis of static and dynamic deformation behavior, as well as on the spatial distribution of contact stresses in the tire–road interface. The approach allows for capturing the influence of varying load levels and inflation pressure on the tire's mechanical response.

The obtained rheological parameters provide a foundation for constructing numerical models of the tire–road contact system, enabling accurate prediction of contact stresses, deformation patterns, and dynamic loading during vehicle operation. The proposed methodologies reduce experimental complexity and improve parameter accuracy, offering practical value for tire design, optimization of vehicle dynamics and transport engineering applications.

**Keywords:** pneumatic tire, tire–road contact, wheel mover, rheological model, elastic–viscous–plastic behavior, deformation, stiffness, contact stress, numerical modeling, vehicle dynamics.

## INTRODUCTION

The efficiency of modern vehicles is largely determined by the ability of their pneumatic-wheel movers to provide stable contact with the road or soil environment under various load conditions. In real conditions, this contact is formed as a result of the complex combination of elastic, plastic, viscoelastic and inertial processes that occur both in the tire structure and in the base layer. Therefore, scientific research aimed at quantitatively describing the mechanisms of deformation and distribution of contact stresses plays the key role in the further development of mobile machinery, road construction and transport engineering [1–5].

The need for improving tire-road analysis methods is increasing due to the transition to intelligent traffic control systems, increasing demands for energy efficiency and tire durability, and the emergence of new composite materials. These materials are characterized by nonlinear and strain rate-dependent properties [3, 6]. This requires the use of models that can correctly reproduce the rheological behavior of rubber-cord structures not only in static but also in transient loading regimes.

At the same time, the effectiveness of such models is directly determined by the reliability of the initial parameters that characterize the mechanical state of the tire material. In particular, the elastic moduli of various structural layers, viscous drag factors, hysteresis loss characteristics, and internal friction parameters need to be clarified using experimental methods consistent with real operating conditions.

Thus, research aimed at improving approaches to determining these parameters has both fundamental and applied significance for increasing the accuracy of predicting the force interaction of the wheel mover with the supporting surface.

## PURPOSE OF THE PAPER

To develop and substantiate methods for determining elastic, viscous and elastic-plastic parameters of pneumatic tire rheological models on based on experimentally obtained dependences of their deformations and contact interaction with the supporting surfaces of the movement of earth-moving machines.

## RESEARCH RESULTS

The processes research of dynamic interaction of the pneumatic wheeled vehicle with the supporting surface undergoing elastic-plastic deformations requires the use of adequate physical and mechanical models capable of reproducing the rheological complexity of the tire rubber-cord structure. In most theoretical approaches, a pneumatic tire is represented as the weightless ring with an outer radius equal to its free radius $r_0$. This ring is conventionally connected to the hub by continuous set of radially oriented deformable elements that simulate the reaction of the rubber-cord shell. These elements form various compositions of basic rheological elements – Hooke, Newton and Saint-Venant [6–8], which allows modelling the elastic, viscous and elastic-plastic properties of the tire in the wide range of loads.

The accuracy of reproducing the real mechanical properties of the pneumatic tire significantly depends on the reliability of the initial parameters of its material and structural model. These include the modulus of elasticity, the factor of viscous resistance and the value of internal dry friction. In this regard, the development of analytical and experimental methods for determining these characteristics is the separate scientific direction, directly related to the problems of optimizing wheel mover designs and predicting their contact behavior.

The determination method for deformation modulus of the pneumatic tire based on the parameters of its normal elasticity obtained during static tests was developed in the paper [6], that based on a three-element model. In particular, analytical dependencies were established that link the deformation modulus with the tire radial stiffness. Analysis of the results shows a significant dependence of these parameters on the magnitude of the normal load $P_Z$ on hub. This indicates the nonlinear nature of the deformation of the rubber cord shell even at low speeds of load application.

The specified dependence of tire parameters on the level of external load necessitates the use of dynamic models of the pneumatic wheeled vehicle with parameters varying in time and space. This complicates the construction of analytical solutions for describing the processes of its interaction with an uneven and deformable supporting surface, which in turn requires the use of numerical methods and algorithms of multiparameter identification.

In the first approximation, it is advisable to represent the pneumatic tire in the form of the linear Kelvin–Voigt viscoelastic element model [7], in which the elements deform exclusively in the radial direction. Let us consider the mode of tire static loading by the normal force $P_Z$ on the rigid, flat and non-deformable horizontal support surface. Within this mode, the rate of radial deformation $V_Z$ is so small that the viscous properties of the material practically do not manifest themselves, and the interaction is reduced to a purely elastic response (Fig. 1). Under such conditions, the radial force in the model element is determined by expression

$$dP = C_P \cdot \Delta_P ,$$

where $C_P$, $\Delta_P$ – stiffness and deformation in the tire radial direction of the tire.

Let us decompose the radial force $dP$ into vertical $dR$ and horizontal $dF$ components – $dR = dP\cos\varphi$ and $dF = dP\sin\varphi$, then under equilibrium conditions, we will have

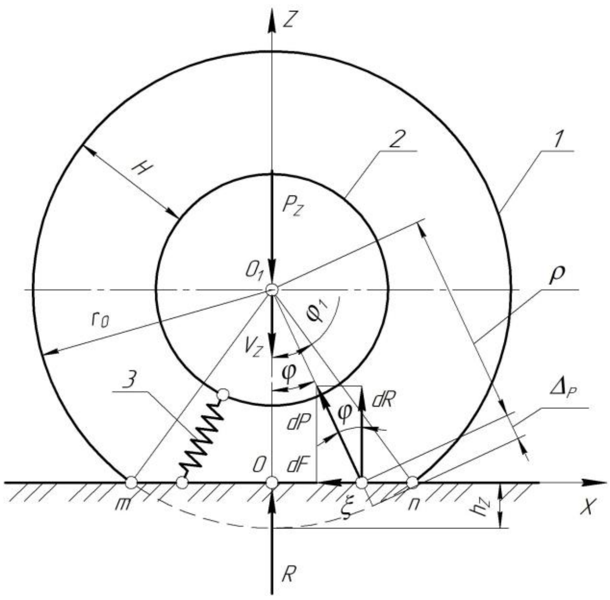


**Fig. 1.** Tire loading diagram during static testing: 1 – rubber cord casing; 2 – hub; 3 – elastic element (Hook element)

$$\sum dR = \sum dP \cos\varphi = R = P_Z; \quad \sum dF = \sum dP \sin\varphi = 0. \quad (1)$$

Taking into account that $\Delta_P = r_0 - \rho = r_0\left(1 - \frac{\cos\varphi_1}{\cos\varphi}\right)$ taking into account expression (1), we obtain

$$P_Z = 2C_P r_0 \int_0^{\varphi_1}\left(1 - \frac{\cos\varphi_1}{\cos\varphi}\right)\cos\varphi d\varphi,$$

where $\rho$ – current radius of the pneumatic tire in the contact zone with the support surface, measured at the current angle $\varphi$; $\varphi_1$ – half the contact angle.

After performing integration and transformation, we will have

$$P_Z = 2C_P r_0\left(\sin\varphi_1 - \varphi_1 \cdot \cos\varphi_1\right), \quad (2)$$

where $\varphi_1 = \arccos\left[(r_0 - h_Z)/r_0\right]$.

To establish the relationship between the $C_P$ value and the parameters of the pneumatic tire, we will use the experimental dependence

$$h_Z = K_Z P_Z^{3/4} \big/ (1 + p_W), \quad (3)$$

where $K_Z$ – factor constant for given elastic element; $p_W$ – air pressure in the pneumatic tire.

The practical application convenience of expression (3) is determined by the presence of only one factor, constant for the pneumatic tire of the wheel mover, in the entire operating range of changes in internal air pressure $p_W$. This allows for the rapid accumulation of experimental data. The factor $K_Z$ can be determined, for example, by the results of measuring the normal tire deflection $h_Z$:

1) at two values of normal hub load $P_{Z1}$, $P_{Z2}$ and one arbitrary value of air pressure

$$K_Z = \Delta h'_Z (1 + p_W)\left(P_{Z2}^{3/4} - P_{Z1}^{3/4}\right);$$

2) at two values of air pressure $p_{W1}$, $p_{W2}$ and one constant value of normal hub load

$$K_Z = \Delta h''_Z (1 + p_{W1})(1 + p_{W2}) \big/ P_Z^{3/4}(p_{W1} - p_{W2}),$$

where $\Delta h'_Z$ – change in normal tire deflection when the normal hub load increases from $P_{Z1}$ to $P_{Z2}$, i.e. $\Delta h'_Z = h'_{Z2} - h'_{Z1}$; $\Delta h''_Z$ – change in normal tire deflection when the air pressure decreases from $p_{W1}$ to $p_{W2}$, i.e. $\Delta h''_Z = h''_{Z2} - h''_{Z1}$.

Let us represent expression (3) in the form $P_Z = P_Z(h_Z)$

$$P_Z = h_Z^{4/3}(1 + p_W)^{4/3} \big/ K_Z^{4/3}$$

or taking into account that $h_Z = r_0(1 - \cos\varphi_1)$

$$P_Z = \left[r_0(1 - \cos\varphi_1)(1 + p_W)/K_Z\right]^{4/3}. \quad (4)$$

Equating the right-hand sides of expressions (2) and (4), after transformations we obtain

$$C_P = \frac{r_0^{1/3}}{2}\left[\frac{(1 + p_W)}{K_Z}\right]^{4/3}\frac{(1 - \cos\varphi_1)^{4/3}}{(\sin\varphi_1 - \varphi_1\cos\varphi_1)}. \quad (5)$$

Let us analyze the right-hand side of expression (5). To do this, in a graphical form (Fig. 2, *a*) we will establish the relationship between the quantities $(1-\cos\varphi_1)^{4/3}$ and $(\sin\varphi_1-\varphi_1\cos\varphi_1)$. As can be seen, the graphical dependence (1) has the weakly nonlinear character and can be quite correctly approximated by the linear dependence (2):

$$(1-\cos\varphi_1)^{4/3}=B_1(\sin\varphi_1-\varphi_1\cos\varphi_1). \quad (6)$$

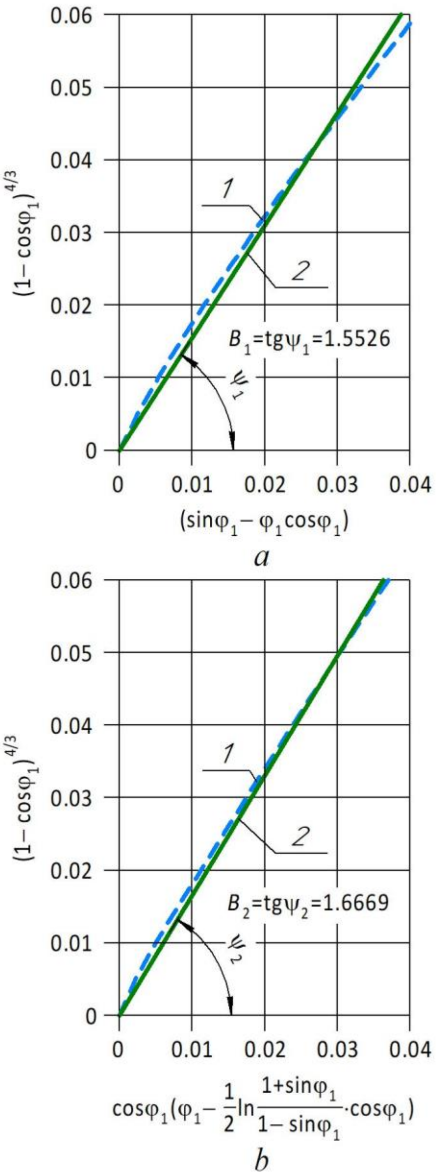


**Fig. 2.** Determination schemes of approximate dependence: *a* – $f(\sin\varphi_1-\varphi_1\cos\varphi_1)$;

*b* – $f\left[\cos\varphi_1\left(\varphi_1-\frac{1}{2}\ln\frac{1+\sin\varphi_1}{1-\sin\varphi_1}\cdot\cos\varphi_1\right)\right]$

The value of the slope factor $B_1$ is determined by the least squares method and is in the range of the maximum possible change $h_Z$ $(0\le h_Z\le 0{,}117r_0)$ $B_1=1{,}5526$ in the correlation factor $R=0{,}995$.

Substituting the right-hand side of dependence (6) into expression (5), we finally write

$$C_P=\frac{B_1 r_0^{1/3}}{2}\cdot\left[\frac{(1+p_W)}{K_Z}\right]^{4/3}. \quad (7)$$

Thus, the obtained expression (7), on the one hand, establishes a relationship between the external dimensions of the tire, normal stiffness and internal air pressure, and on the other hand, with the radial stiffness of the elastic element of the adopted pneumatic tire model (see Fig. 1).

When use the deformation laws in the problems of the rotating element dynamics, the determination of the tire deformation factors in the loading zone ($nO$) – $K_1$ and in the unloading zone ($Om$) – $K_2$ [8] can be performed similarly to that considered above.

During static testing of the pneumatic tire by normal hub load $P_Z$, taking into account the law of its deformation – $\sigma=K_1\Delta_P$, when the magnitude of the radial deformation of the tire $\Delta_P$ does not exceed its critical value $\Delta_{PK}$, the following expression will be valid

$$P_Z=2\int_0^{\varphi_1}\sigma B_{tr}\rho\cos\varphi d\varphi=2K_1B_{tr}r_0^2\cos\varphi_1\times$$
$$\times\left(\varphi_1-\frac{1}{2}\ln\frac{1+\sin\varphi_1}{1-\sin\varphi_1}\cdot\cos\varphi_1\right), \quad (8)$$

where $\sigma$ – normal stress in the contact zone; $B_{tr}$ – tire tread width.

Equating the right-hand sides of expressions (4) and (8), after transformations we obtain

$$K_1 = \frac{\left(1+p_W\right)^{4/3}}{2B_{tr}r_0^{2/3}K_Z^{4/3}} \times$$
$$\times \frac{\left(1-\cos\varphi_1\right)^{4/3}}{\cos\varphi_1\left(\varphi_1 - \frac{1}{2}\ln\frac{1+\sin\varphi_1}{1-\sin\varphi_1}\cos\varphi_1\right)} . \quad (9)$$

The dependence $\left(1-\cos\varphi_1\right)^{4/3} = f\left[\cos\varphi_1\left(\varphi_1 - \frac{1}{2}\ln\frac{1+\sin\varphi_1}{1-\sin\varphi_1}\cos\varphi_1\right)\right]$ is presented in Fig. 2, *b*. As can be seen, the dependence (1) in graphical form also has the weakly nonlinear character and can be approximated by the linear dependence (2)

$$B_2\left[\cos\varphi_1\left(\varphi_1 - \frac{1}{2}\ln\frac{1+\sin\varphi_1}{1-\sin\varphi_1}\cos\varphi_1\right)\right]. \quad (10)$$

The slope value $B_2$ is determined by the least squares method and is in the range of the maximum possible change $0 \le h_Z \le 0{,}117 r_0$ $B_2 = 1{,}6669$ at the correlation factor of 0.995.

Taking into account expressions (9) and (10), we finally write

$$K_1 = \frac{B_2}{2B_{tr}r_0^{2/3}} \cdot \left[\frac{\left(1+p_W\right)}{K_Z}\right]^{4/3} . \quad (11)$$

Taking into account expressions (7) and (11), we establish the relationship between the quantities $C_P$ and $K_1$

$$C_P = B_1 K_1 B_{tr} r_0 / B_2 = B_3 K_1 , \quad (12)$$

where $B_3 = B_1 B_{tr} r_0 / B_2$ – factor, constant for the pneumatic tire.

Thus, two complementary methods for determining the elastic parameters (radial stiffness $C_P$) of the tire rheological model in the form of the linear Kelvin–Voigt element model are considered based on:

1) experimental dependences of the normal tire deformation on its normal load $h_Z = h_Z\left(P_Z\right)$;

2) experimental dependences of the deformation distribution law in the contact zone of the tire with the non-deforming supporting surface $\sigma = \sigma\left(\Delta_P\right)$.

Using the second method is more appropriate, since to determine the elastic parameters of the rheological model, it is not necessary to conduct tests on crimping stands. It is only to experimentally determine the radial tire deformation $\Delta_P$ and the normal stresses $\sigma$ in the zone of its contact with the supporting surface. Let us move now from the linear Kelvin–Voigt element model to a three-element rheological tire model with parallel fundamental elements of Hooke, Newton and Saint-Venant, the elementary components of which deform in the radial direction.

This model reflects the elastic properties of the tire and takes into account the presence of both types of internal friction (viscous and dry) in the rubber cord casing materials [8–12]. The parameters of the rheological tire model (deformation modulus $E_1$, viscosity factor $\mu_1$ and factor $\nu$, which shows what the deformation modulus part of the elastic element is made up of contact stresses due to constant internal friction) can be determined from the results of its static and dynamic tests at different internal air pressures [13].

When obtaining the characteristics of normal stiffness $P_Z = P_Z(h_Z)$, i.e. during static tests of the pneumatic tire of the non-rotating wheeled vehicle $\omega_K = 0$, and damped vibrations $h_Z = h_Z(t)$, i.e. during dynamic tests on the non-deforming support surface, the front (*nO*) and rear (*Om*) contact tire zones with the support surface are symmetrical, and their rheological models work the same.

The plane stress-strain state in contact of the pneumatic tire with the non-deforming support surface at $\omega_K = 0$ is determined

$$\sigma = \varepsilon_1 E_1 + \dot{\varepsilon}_1 \mu_1 + \nu E_1 \operatorname{sgn} \varepsilon_1 . \quad (13)$$

Using the equilibrium equation $P_Z = 2\int_0^{\varphi_1} \sigma_1 \cos\varphi B_{tr} \rho d\varphi$, will be

$$P_Z = 2B_{tr} r_0 E_1 \cos\varphi_1 \left[ \frac{\lambda}{2\cos\varphi_1} \cdot \frac{r_0}{H} \pm \pm \frac{1}{2} \cdot \frac{\mu_1 \dot{h}_Z}{E_1 H} \ln \frac{(1+\sin\varphi_1)}{(1-\sin\varphi_1)} \pm v\varphi_1 sngh_Z \right], \quad (14)$$

where $h_Z$ – tire normal deformation; $\dot{h}_Z$ – rate of tire normal deformation; $\lambda$ – dimensionless multiplier; $\varphi_1$ – central angle of half the contact length of the tire with the non-deforming support surface.

The expression for $\lambda$ has the form

$$\lambda = 2\cos\varphi_1 \left[ \varphi_1 - \frac{1}{2} \ln \frac{(1+\sin\varphi_1)}{(1-\sin\varphi_1)} \times \times \cos\varphi_1 \right] \approx 0,551\varphi_1^{2,88}. \quad (15)$$

Let us represent equation (14) in the following form

$$P_Z = P_{ZE} \pm P_{Z\mu} \pm P_{Z\nu}, \quad (16)$$

where $P_{ZE}$, $P_{Z\mu}$, $P_{Z\nu}$ – fractions of the vertical (normal) load, which is balanced by elastic forces, proportional $\dot{h}_Z$ internal friction forces, and constant internal friction forces.

During static tests of wheel mover tires

$$P_Z = P_{ZE} \pm P_{Z\nu}.$$

Then the dependencies for determining $E_1$ and $\nu$ from (14) will have the form:

$$E_1 = \frac{(P_Z - P_{Z\nu})H}{B_{tr} r_0^2 \lambda}; \quad (17)$$

$$v = \frac{P_{Z\nu}}{2B_{tr} E_1 r_0 \varphi_1 \cos\varphi_1}. \quad (18)$$

Fig. 3 shows the characteristics of static normal stiffness (*a*) and free damped vibrations (*b*) of the tire. It should be borne in mind that the Saint-Venant element of the tire rheological model has a certain uncertainty in the absence of displacements (at it can take values from to including zero). Therefore, equation (13) is applicable only to points at which the tire is deformed or its deformation is restored.

Static normal load $P_Z$ is perceived by elastic forces $P_{ZE}$ and forces of constant internal friction $P_{Z\nu}$. The pneumatic tire has $h_Z$ deformation, and without taking into account internal friction forces — $h_{Z0}$ deformation, which can be calculated or determined graphically by the characteristic of static normal stiffness (Fig. 3, *a*).

With damped vibrations relative to the corresponding position $h_{Z0}$ (Fig. 3, *b*), which are combined by force $P_Z$ with the tire deformation to $h_{Z\max}$ and subsequent rapid release, the equilibrium condition is satisfied

$$P_Z - P_{ZE} \pm P_{Z\nu} \pm P_{Z\mu} \pm P_{Zj} = 0,$$

where $P_{Zj}$ – inertia force of the masses that oscillate together with the tire mass.

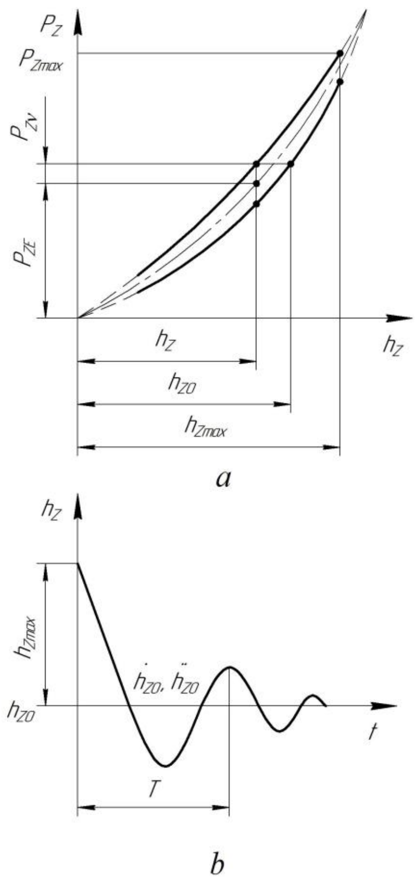


**Fig. 3.** Characteristics of static normal stiffness (*a*) and free damped vibrations (*b*) of the wheel mover tire: *T* – period of oscillations; *t* – time


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At the time corresponding to the deformation $h_{Z0}$

$$P_Z = P_{ZE}\,; \quad P_{Zv} + P_{Z\mu} = P_{Zj}\,,$$

that is, the forces of internal friction are overcome by the inertia force, which is caused by the motion deceleration $\ddot{h}_{Z0}$.

Thus, we write

$$P_{Z\mu} = P_{Zj} - P_{Z\nu}\,; \qquad (19)$$

$$P_{Zj} = P_Z \ddot{h}_Z \,/\, g\,, \qquad (20)$$

where $g$ – acceleration of free fall.

Taking into account expressions (19) and (20), from (14) we obtain

$$\mu_1 = \frac{\left(P_Z \ddot{h}_{Z0} / g - P_{Z\nu}\right) H}{B_{tr} r_0 \dot{h}_{Z0} \ln\left(\dfrac{1+\sin\varphi_1}{1-\sin\varphi_1}\right)\cos\varphi_1}. \qquad (21)$$

Using the dependence (21) to obtain the tire viscosity factor – $\mu_1$ is complicated by the fact that experimental data on the speed $\dot{h}_{Z0}$ and acceleration $\ddot{h}_{Z0}$ at the moment corresponding to the tire normal deformation $h_{Z0}$.

The problem can be solved either by graphical differentiation of the oscillation vibrations, i.e. dependence $h_Z = h_Z(t)$ [6, 13], which introduces the significant error in the determination of $\mu_1$, or by using special sensors (differential links), which complicates experimental studies.

Therefore, the viscosity factor is determined by the equation

$$\mu_1 = -\frac{2m}{TL_C} \cdot \ln\frac{a_{i+1}}{a_i}\,, \qquad (22)$$

where $m$ – reduced mass, including the tire mass – $m_t$ and the rim – $m_r$, kg; $T$ – period of oscillations, s; $L_C$ – contact length at $h_{Z0}$, m; $a_i$, $a_{i+1}$ – any two consecutive amplitudes of damped oscillations, m.

The elementary volumes deformability of the support surfaces with which the wheeled mover tire interacts is quite correctly reflected by the mechanical model of the Kelvin–Voigt viscoelastic element, to which the rheological equation corresponds

$$\sigma = \varepsilon_2 E_2 \pm \dot{\varepsilon}_2 \mu_2\,. \qquad (23)$$

Determination methods of the rheological model parameters of the deformable support surface (deformation modulus and viscosity factor) are currently not sufficiently developed and it is possible to recommend the use of only some experimental data given in the papers [6, 10, 12–16]. In this regard, the authors have developed analytical methods and the methodology for experimentally determining the parameters of the rheological model for the deformable support surface.

Deformation of the support surface in the contact zone with the tire occurs, as with the latter, in the radial direction relative to its center. Then, when rolling a non-deformable wheeled mover, the support surface will deform in the directions $\xi_1 K_1$ ($\xi_2 K_2$) according to the scheme in Fig. 4, and the relative deformations $\varepsilon_2$ and deformation rates $\dot{\varepsilon}_2$ of its elementary volumes (within the angle $d\varphi$) will be determined by equations:

for loading zone (*nO*)

$$\varepsilon_{21} = \frac{\left(A + r_0\cos\varphi_1\right)\left(1 - \cos\varphi_1 / \cos\varphi\right)}{A}\,;$$

$$\dot{\varepsilon}_{21} = -\frac{\omega_W \varepsilon_{21} \cos\varphi_1 tg\varphi}{\cos\varphi - \cos\varphi_1}\,;$$

for unloading zone (*Om*)

$$\varepsilon_{22} = \frac{\left(A + r_0\cos\varphi_1\right)\left(1 - \cos\varphi_2 / \cos\varphi\right)}{A + r_0\left(1 - \cos\varphi_1\cos\varphi_2\right)\cos\varphi_1}\,;$$

$$\dot{\varepsilon}_{22} = \frac{-\omega_W \varepsilon_{22} \cos\varphi_2 tg\varphi}{\cos\varphi - \cos\varphi_2}.$$

Equation (23) for the corresponding contact areas of the non-deformable wheeled mover with the deformable support surface (*nO* – «21» index, *Om* – «22» index) have the form:

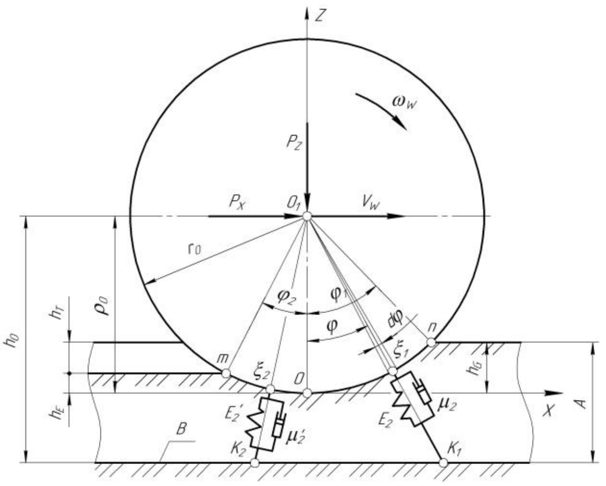

**Fig. 4.** Force interaction scheme of the non-deformable wheeled mover with deformable support surface in the driven mode

$$\sigma_{21} = \varepsilon_{21} E_2 + \dot{\varepsilon}_{21} \mu_2 ;$$
$$\sigma_{22} = \varepsilon_{22} E_2' - \dot{\varepsilon}_{22} \mu_2' ,$$

where $E_2'$ and $\mu_2'$ parameters are determined by equations:

$$E_2' = k_{ПГ} \cdot E_2 ;$$
$$\mu_2' = k_{ПГ} \cdot \mu_2 .$$

The elasticity factor of the deforming support surface $k_E$ is determined by equation

$$k_E = \frac{h_E}{h_G} = \frac{1 - \cos\varphi_2}{1 - \cos\varphi_1} , \qquad (24)$$

where $h_E$, $h_G$ – elastic and total deformations of the deformable support surface (soil); $\varphi_1$, $\varphi_2$ – corresponding contact angles of the non-deformable wheeled mover with the deformable support surface.

Under the condition of uniform distribution of contact stresses over the tread width $B_{tr}$, the equilibrium equations of the non-deformable wheeled mover in the driven mode of its power load [12] have the form:

$$P_Z = B_{tr} r_0 \left( \int_0^{\varphi_1} \sigma_{21} \cos\varphi d\varphi + \int_0^{\varphi_2} \sigma_{22} \cos\varphi d\varphi \right) ; (25)$$

$$P_X = B_{tr} r_0 \left( \int_0^{\varphi_1} \sigma_{21} \sin\varphi d\varphi - \int_0^{\varphi_2} \sigma_{22} \sin\varphi d\varphi \right) . (26)$$

As a result of (25) and (26) joint solution, the expressions for the definition of $E_2$ and $\mu_2$ are obtained:

$$E_2 = \frac{P_X + P_Z \cdot C / D}{B_{tr} r_0 \left( A + r_0 \cos\varphi_1 \right) \left( F + CK / D \right)} ; \qquad (27)$$

$$\mu_2 = \frac{r_0}{V_W} \cdot \frac{P_Z / \left[ B_{tr} r_0 \left( A + r_0 \cos\varphi_1 \right) \right] - E_2 K}{D} , (28)$$

where

$$C = \frac{\cos\varphi_1 \left( tg\varphi_1 - \varphi_1 \right)}{A} + $$
$$+ k_E \frac{\cos\varphi_2 \left( tg\varphi_2 - \varphi_2 \right)}{A + r_0 (1 - \cos\varphi_1 \cos\varphi_2) \cos\varphi_1} ;$$

$$D = \frac{\cos\varphi_1 \ln(\cos\varphi_1)}{A} - $$
$$- k_E \frac{\cos\varphi_2 \ln(\cos\varphi_2)}{A + r_0 (1 - \cos\varphi_1 \cos\varphi_2) \cos\varphi_1} ;$$

$$F = \frac{(\cos\varphi_1 - 1)^2}{2A} - $$
$$- k_E \frac{(\cos\varphi_2 - 1)^2}{2 \left( A + r_0 (1 - \cos\varphi_1 \cos\varphi_2) \cos\varphi_1 \right)} ;$$

$$K = \frac{\sin\varphi_1 - \varphi_1 \cos\varphi_1}{A} + $$
$$+ \frac{\sin\varphi_2 - \varphi_2 \cos\varphi_2}{A + r_0 (1 - \cos\varphi_1 \cos\varphi_2) \cos\varphi_1} .$$

According to (27) and (28) equations of the rolling process of the non-deformable wheeled mover in the driven mode of its force loading on the deformable support surface, to determine the parameters of its rheological model in the form of Kelvin–Voigt element, it is necessary to experimentally determine the longitudinal force on the wheel axis, the contact angles of the non-deformable wheeled mover in the loading (*nO*) and unloading (*Om*) zones, the total and elastic deformations of the support surface, as well as the thickness of the soil layer undergoing deformations.

## CONCLUSIONS

The parameters of rheological models (stiffness, modulus of deformation, viscosity and dry friction factors) significantly depend on the normal load and internal air pressure, which has been confirmed experimentally.

Determination methods of radial stiffness tire parameters can be implemented based on: normal tire deformation under static loads and the law of deformation distribution in the tire contact zone with the supporting surface. The second method is practically simpler and less resource-intensive.

The three-element model (Hook–Newton–Saint-Venant) provides the more detailed reproduction of internal processes in the tire, taking into account both viscous and dry components of internal friction. The obtained dependencies make it possible to comprehensively establish the relationship between the geometric tire parameters, air pressure and its elastic characteristics.

The proposed analytical formulation creates the basis for building numerical models of wheel–soil interaction, in particular for calculating dynamic loads, contact stresses and forming loading/unloading zones.

The application of the determination method for radial stiffness by the distribution of deformation in the contact patch allows:

– to reduce 40–60% experimental time;

– to reduce the error in determining stiffness from 12% to 3–5%;

– to obtain parameters for elastic-viscous-plastic models without the need for full static tire loading.

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## Визначення реологічних параметрів системи «шина–дорога»

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**Анотація.** Ефективність і безпека сучасних колісних транспортних засобів значною мірою визначаються надійною взаємодією пневматичних шин з опорною дорожньою або ґрунтовою поверхнею за різних режимів навантаження. Ця взаємодія формується під впливом складного поєднання пружних, в'язких і пластичних деформаційних процесів, що відбуваються як у конструкції шини, так і в опорному шарі поверхні контакту. Для точного опису цих процесів необхідні розробка та застосування вдосконалених реологічних моделей, здатних відтворювати нелінійну та часозалежну поведінку матеріалів шини.

У роботі запропоновано аналітичні та експериментальні методики визначення реологічних параметрів пневматичних шин, включаючи жорсткість, модуль деформації, коефіцієнт в'язкого опору та характеристики внутрішнього тертя. Методи базуються на аналізі статичної та динамічної поведінки деформацій шини, а також на дослідженні просторового розподілу контактних напружень у зоні взаємодії шини з опорною поверхнею. Підхід дозволяє враховувати вплив різного рівня навантаження та внутрішнього тиску повітря на механічну реакцію шини.

Отримані реологічні параметри створюють основу для побудови числових моделей системи «шина–дорога», що забезпечує точне прогнозування контактних напружень, закономірностей деформування та динамічних навантажень під час експлуатації транспортних засобів. Запропоновані методики зменшують експериментальну складність та підвищують точність визначення параметрів, маючи практичне значення для конструкції шин, оптимізації динаміки руху та інженерії транспорту.

**Ключові слова:** пневматична шина, контакт шина–дорога, колісний рушій, реологічна модель, пружно-в'язкопластична поведінка, деформація, жорсткість, контактні напруження, числове моделювання, динаміка руху.