

Determination of the optimal average fluid velocity in the sections of a variable-diameter collecting drainage pipeline

Oleksandr Kravchuk¹, Olga Kravchuk²

¹ Kyiv National University of Construction and Architecture,
31 Povitrianykh Syl Avenue, Kyiv, Ukraine, 03037

² National Transport University,
1 M. Omelianovycha-Pavlenka Str., Kyiv, Ukraine, 01010
¹ kravchuk.oa2@knuba.edu.ua, <https://orcid.org/0000-0001-6578-8896>,
² olgakravchuk56@gmail.com, <https://orcid.org/0000-0003-2616-5455>

Received 24.10.2025; Accepted 25.11.2025

<https://doi.org/10.32347/gbdmm.2025.106.0101>

Abstract. An analysis of the operating conditions of pressure collecting drainage pipelines has shown that such conduits are typically considered as channels with a constant cross-section along their length, operating under variable discharge. In the initial sections of these pipelines, the average velocities of the main flow are relatively low (below permissible limits), which often leads to the deposition and accumulation of significant amounts of soil particles in the initial sections. In the final sections, the average flow velocity becomes excessively high, causing an increase in head losses. Therefore, the problem of determining the optimal value of the average fluid velocity along the length of a collecting pipeline is relevant.

Taking into account the above, the purpose of the present study is to develop a calculation methodology for determining the optimal average fluid velocity along the length of pressure collecting drainage pipelines with a variable (increasing) cross-section.

In this work, based on a dimensionless analysis of a system of differential equations describing fluid flow with variable discharge in pressure collecting drainage pipelines with an increasing cross-section, a methodology for calculating the optimal average fluid velocity is proposed. During the analysis, the second term in the differential equation of variable discharge, representing head losses associated with the effect of fluid attachment along the pipeline length, was neglected, as it has negligible influence on the final computational results. The study considers the concept of a conditionally infinite collecting pipeline or a finite pipeline with an infinitely permeable lateral surface. The key parameters determining the change in flow rate along the collecting drainage pipeline are taken as the

collector resistance coefficient ζ_{col} and the generalized drainage collector parameter A_{col} , which accounts for its structural and filtration characteristics.

As a result, a simple and practical method has been obtained for determining the maximum average velocity in the final section of the collector, taking into account the variable diameter and the non-uniform inflow distribution along its length.

Keywords: collecting drainage pipeline, hydraulic friction factor, filtration coefficient, filtration resistance, variable flow rate.

INTRODUCTION

Rational use, protection, and restoration of water resources are among the most important tasks for the sustainable development of agriculture, industry, and urbanized territories. Under conditions of climate change, increasing frequency of extreme precipitation events, rising groundwater levels, and intensified anthropogenic pressure on natural ecosystems, the demand for improved drainage and water-regulation systems continues to grow [1, 2]. This is particularly important for Ukraine, where extensive agricultural lands and populated areas depend on the effective operation of reclamation systems, structures, and subsurface water intakes [3].

Most existing drainage systems in Ukraine were designed using typical mid-20th-century solutions, employing ceramic or asbestos-cement pipes of fixed diameter. Such designs no longer meet modern requirements for ener-

gy efficiency, reliability, and environmental safety. In the initial sections of collecting pipelines, the water discharge is relatively small. Consequently, the average flow velocities in these sections are low, leading to the deposition and accumulation of soil particles inside the pipeline. Over time, the accumulated sediment obstructs water movement (up to complete blockage of the cross-section and failure of the drain). Restoring the operation of such drains requires special flushing procedures, which significantly increase the cost of maintenance and construction of these systems. Conversely, in the final sections of drainage pipelines, the water discharge is considerably higher. With a constant cross-section, the average flow velocity becomes excessive, resulting in substantial head losses along the pipeline. As a result, many existing facilities currently operate under overload conditions, and the efficiency of groundwater lowering in certain areas decreases significantly.

Under modern conditions, when the modernization of drainage systems must be not only technically but also economically feasible, there is a growing need to develop new engineering solutions that ensure stable operation of collector – drainage networks while minimizing material and energy costs. One promising approach is the use of collecting drainage pipelines with a variable (increasing) cross-section along their length. This design concept makes it possible to account for changes in discharge caused by lateral drain inflows or filtration inflow, ensure a more uniform velocity distribution, and reduce hydraulic losses.

However, the correct design of such pipelines requires improved hydraulic calculation methods that consider flow variability along the pipeline and transitional flow regimes. Traditional methods based on the assumptions of constant cross-section and steady flow do not allow accurate determination of head losses, optimal diameter, or installation depth for variable-diameter pipelines. As a result, design errors often occur, leading to excessive material consumption, reduced system efficiency, or uneven distribution of the filtration flow.

A considerable number of scientific studies in Ukraine and abroad have been devoted to

improving the efficiency of collecting drainage pipelines [4-7]. The latest developments in this field focus on increasing the operational efficiency of reclamation systems [8, 9] and optimizing the parameters of drainage pipelines to maintain stable soil water regimes [10, 11].

However, when describing the parameters of fluid flow in the investigated pipelines, nearly all authors employed certain assumptions that were not always sufficiently justified. This is primarily due to the complexity of solving the initial mathematical model – the system of differential equations describing fluid flow with variable discharge [12]. In particular, it was often assumed that drainage pipelines operate under a constant discharge ($q = \text{const}$) [13], or that the discharge varies uniformly along the length ($q = dQ/dx = \text{const}$) [14]. Many studies focused exclusively on the hydraulic aspects, while the filtration characteristics of the system were either not considered at all or were taken into account insufficiently [15].

Recent research has also focused on integrating hydraulic calculations into digital management systems for drainage networks. Several publications [16, 17] describe the use of BIM (Building Information Modeling) and GIS (Geographic Information System) technologies for constructing digital models of subsurface drainage systems, enabling flow-regime calculations based on actual monitoring data. However, their effective application requires reliable analytical relationships that can serve as a basis for model verification and automated calculation systems.

Thus, the conducted analysis shows that although individual aspects of hydraulic calculations for drainage pipelines have been studied in sufficient detail, comprehensive methodologies that account for variable cross-sectional geometry, non-uniform filtration inflow, and real operational conditions still require further development. This highlights the need for improving existing approaches and developing new analytical relationships for determining the hydraulic characteristics of collecting drainage pipelines with variable cross-section, particularly the maximum average velocity in the final section of the collector. The obtained results can be used to refine design

approaches for the reconstruction and new construction of drainage systems, water-intake structures, and wastewater networks, as well as to enhance their energy efficiency, durability, and operational reliability.

PURPOSE OF THE PAPER

The purpose of this study is to develop a calculation methodology for determining the optimal average fluid velocity in the cross-sections of pressure collecting drainage pipelines with a variable (increasing) cross-section, operating under conditions of a horizontal groundwater level.

RESEARCH RESULTS

During the design and operation of collecting drainage pipelines of reclamation systems and various types of radial well intakes, the primary criterion for selecting the law governing the variation of the pipe diameter is the requirement to maintain a constant average flow velocity along the entire length of the pipeline. In all cases, this velocity must not fall below the non-silting (self-cleaning) threshold. At the same time, for collecting drainage pipelines, there always exists a maximum average flow velocity in the final cross-section. Its value for a given pipeline depends on the ratio between the structural and filtration characteristics of the drain and the surrounding soil. It is evident

that if a constant average velocity is to be ensured along the length of the conduit, the pattern of pipe diameter variation will also depend on the intensity of water inflow at different sections of the drainage pipeline

$$(q = \frac{dQ}{dx} = V \frac{d\Omega}{dx}).$$

This study considers the case of a horizontal pressure collecting drainage pipeline operating without transit under conditions of non-uniform variation of specific discharge along its length. The schematic operation of such a collector is shown in Fig. 1.

In the analysis and description of the operating characteristics of collecting drainage pipelines, a system of differential equations is typically used, consisting of the equation of motion for a variable-mass fluid and the continuity equation expressed as a relationship describing the inflow of fluid into the pipeline, in the form of a modified filtration equation [18, 19]:

$$\frac{dh}{dx} + \frac{2V}{g} \frac{dV}{dx} + \frac{\lambda_{col} V^2}{2gD} = 0 \quad (1)$$

$$q = \frac{dQ}{dx} = V \frac{d\Omega}{dx} = k_f \frac{H-h}{F}, \quad (2)$$

where V is the specified constant average velocity in the flow cross-section along the drainage pipeline; D and Ω are the diameter and cross-sectional area of the pipe, which vary along the length; λ_{col} is the hydraulic

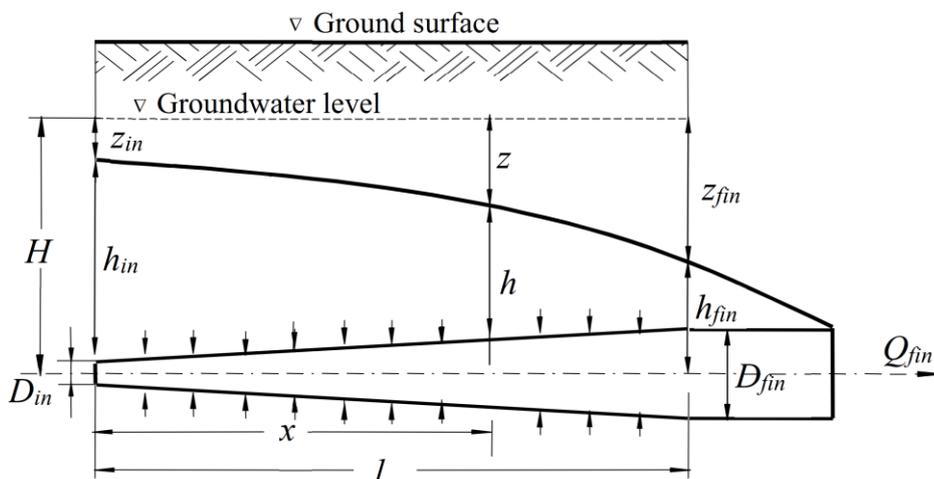


Fig. 1. Scheme of the variable-diameter collecting drainage pipeline operation

friction factor; k_f is the filtration coefficient of the surrounding soil; $H - h = z$ is the head difference driving the inflow of fluid from the surrounding medium into the drain; \bar{F} is the filtration resistance of the “soil–drain” system.

As shown in several studies [20, 21], for $V = \text{const}$ and under conditions of fluid inflow in filtration mode, the second term in equation (1) can be neglected. It is also assumed that $dh = -dz$. Taking this into account, equation (1) can be written as follows:

$$-\frac{dz}{dx} + \frac{\lambda_{col} V^2}{2gD} = 0. \quad (3)$$

For further analysis, by introducing new dimensionless variables:

$$\bar{x} = \frac{k_{fin} x}{\Omega_{fin} \bar{F}} \sqrt{\frac{z_{fin}}{g}}, \quad \bar{z} = \frac{z}{z_{fin}}, \quad \bar{D} = \frac{D}{D_{fin}}, \quad (4)$$

where D_{fin} , Ω_{fin} is the diameter and cross-sectional area of the collecting pipeline at its final section, the original system of equations is reduced to the following form:

$$\frac{d\bar{z}}{d\bar{x}} = \zeta_{l_{col},fin} A_{col} \bar{V}^2 \frac{1}{\bar{D}} \quad (5)$$

$$2\bar{V}\bar{D} \frac{d\bar{D}}{d\bar{x}} = \bar{z}, \quad (6)$$

where $\zeta_{l_{col},fin} = \lambda_{col} \frac{l}{D_{fin}}$ is the resistance

coefficient of the collecting drainage pipeline, calculated based on the diameter at its final section; $A_{col} = \frac{1}{2\bar{x}_{fin}} = \frac{\Omega_{fin} \bar{F}}{2k_{fin} l} \sqrt{\frac{g}{z_{fin}}}$ is the

generalized parameter of the collecting drain, which accounts for its structural and filtration

characteristics; $\bar{V} = \frac{V}{\sqrt{gz_{col}}} = \text{const}$.

A detailed analysis of the system of differential equations (5) and (6) was carried out in [22]. According to this analysis, the relative diameter of the pipeline, varying along the length of the collector was determined by the following relationship:

$$\bar{D} = \sqrt[3]{\left(3\zeta_{l_{col},fin} A_{col} \bar{V}^2 \bar{x}\right)^2} \quad (7)$$

$$\text{or} \quad \bar{D} = \sqrt[3]{\left(\frac{3}{2}\zeta_{l_{col},fin} \bar{V}^2 \frac{x}{l}\right)^2}. \quad (8)$$

At the final section, for $x = l$ and $\bar{D}_{fin} = 1$, in the given relationships, we have:

$$\sqrt[3]{\left(\frac{3}{2}\zeta_{l_{col},fin} \bar{V}^2\right)^2} = 1. \quad (9)$$

As noted above, the minimum constant value of the average fluid velocity in the cross-sections along the length of the drain is taken equal to the non-silting velocity ($V_{min} = V_{n.s.}$). This velocity is also referred to as the settling velocity. Its magnitude depends on the characteristics of the soil particles and must not be lower than the critical velocity V_{cr} , which is determined according to the corresponding empirical relationships. The specialized literature [23] provides numerous empirical relationships proposed by different authors for determining this parameter. For example, equation (11), which, in our view, adequately corresponds to the results of experimental studies:

$$V_{cr} = 1,88 \sqrt{\frac{gDV_{h.s.} \rho_p - \rho_w}{\lambda \rho_w}}, \quad (10)$$

where $V_{h.s.}$ is the hydraulic particle size of the transported material; D is the diameter of the pipeline; λ is the hydraulic friction factor for the flow of clean water in the pipeline; ρ_p is the density of the pulp; ρ_w is the density of water.

The maximum average velocity in the final section of a pressure collecting drainage pipeline, given its diameter D_f , is calculated using the following formula:

$$\bar{V}_{max} = \sqrt{\frac{2}{3\zeta_{l_{col},fin}}}. \quad (11)$$

The graph of relationship (11) shown on Fig. 2.

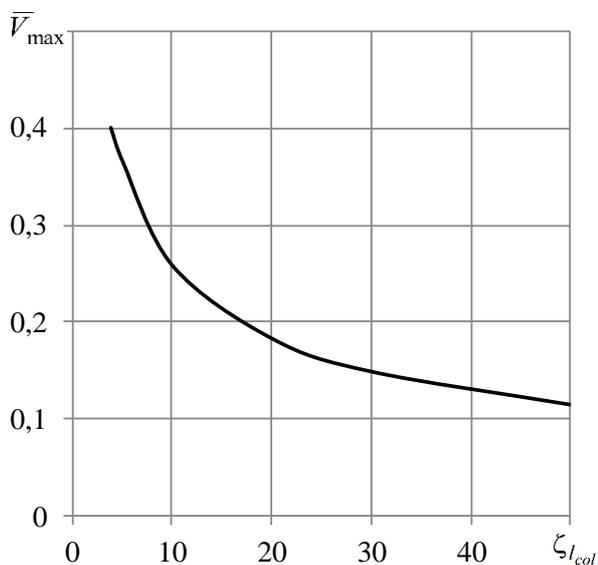


Fig. 2. Dependence of the maximum relative velocity in the final section of the collector \bar{V}_{max} on its resistance coefficient $\zeta_{l_{col}}$

Alternatively, relationship (11) can be expressed in dimensional terms:

$$V_{max} = \sqrt{\frac{2gz_{fin}}{3\zeta_{l_{col}.fin}}}, \text{ m/s.} \quad (12)$$

In the case where the maximum water velocity in the cross-sections of the collector is known due to technological conditions, the maximum diameter at the end of the considered pipeline is determined using the relationship derived from (12):

$$D_{fin} = \frac{3\lambda_{col}lV^2}{2gz_{col}}, \text{ m.} \quad (13)$$

The calculation results obtained using the proposed relationships are satisfactorily confirmed by field measurement data.

CONCLUSIONS

Based on the analysis of differential

equations describing fluid flow in pressure collecting drainage pipelines with a variable (increasing) cross-section, and an examination of their main hydraulic characteristics and structural cross-sectional parameters along the length, a simple and convenient methodology was developed for calculating the maximum average velocity in the final section of a collecting pipeline. The corresponding calculation relationships and supporting graphs were presented.

It has been shown that the magnitude of the maximum allowable velocity significantly depends on the pipeline diameter variation, the intensity of lateral inflow, and the hydraulic resistance. A generalized relationship has been obtained for determining the maximum average velocity in the final section of the collector.

The results of this study contribute to the improvement of the theoretical foundations of hydraulic calculations for collecting drainage systems and can be applied in the design of modern drainage systems, reclamation drains, and underground water intake structures.

REFERENCES

1. **Xie, H., Huang, Y., Chen, Q., Zhang, Y., & Wu, Q.** (2019). Prospects for agricultural sustainable intensification: A review of research. *Land*, 8(11), 157. <https://doi.org/10.3390/land8110157>
2. **Majumder, R.** (2023). Balancing food security and environmental safety: rethinking modern agricultural practices. *Environmental & Experimental Biology*, 21(4), 101-110. <http://doi.org/10.22364/eeb.21.12>
3. **Baliuk, S., Romashchenko, M., & Truskavetskyi, R.** (2018). Problems of environmental risks and perspectives of land reclamation in Ukraine. *AgroChemistry and Soil Science*, 87, 5-10. (in Ukrainian). <https://doi.org/10.31073/acss87-01>
4. **Vlotman, W., Smedema, L., & Rycroft, D.** (2020). *Modern land drainage: planning, design and management of agricultural drainage systems*. London: CRC Press. 502. <https://doi.org/10.1201/9781003025900>
5. **Collins, S., & Houben, G. J.** (2020). Horizontal and radial collector wells: simple tools for a complex problem. *Hydrogeology Journal*, 28,

- 1925-1935. <https://doi.org/10.1007/s10040-020-02120-2>
6. **Cherniuk, V., Kravchuk, O., Fasuliak, V., & Cherniuk, M.** (2024). Improvement of modeling of laminar flows in pressure collector-pipelines. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 120(2), 182-196. <https://doi.org/10.37934/arfmts.120.2.182196>
 7. **Kravchuk, A. M., Chernyshev, D. O., & Kravchuk, O. A.** (2021). Hydraulics of pressure perforated pipelines of treatment facilities of water supply and water disposal systems: monograph. Kyiv: KNUCA. 204. (*in Ukrainian*).
 8. **Gurovich, L., & Oyarce, P.** (2015). New approaches to agricultural land drainage: a review. *Irrigation & Drainage Systems Engineering*, 4(135), 2. <http://dx.doi.org/10.4172/2168-9768.1000135>
 9. **Tkachuk, M. M., Turcheniuk, V. O., Shynkaruk, L. A., & Tkachuk, L. R.** (2021). Ensuring efficiency of water regulation on water management systems with innovative structures. *Bulletin National University of Water and Environmental Engineering*, 4(96), 106-116. (*in Ukrainian*). <https://doi.org/10.31713/vt420219>
 10. **Wu, Z., Guo, C., Yang, H., Li, H., & Wu, J.** (2022). Experimentally based numerical simulation of the influence of the agricultural subsurface drainage pipe geometric structure on drainage flow. *Agriculture*, 12(12):2174. <https://doi.org/10.3390/agriculture12122174>
 11. **Zhang, Q., & Wang, Z.** (2014). Modeling study on fluid flow in horizontal perforated pipes with wall influx. *International Journal of Fluid Mechanics Research*, 6 (14), 556-566. <https://doi.org/10.1615/InterJFluidMechRes.v4.1.i6.80>
 12. **Kravchuk, O. A.** (2021). Particularities of hydraulic calculation of collecting pressure drainage pipelines. *Bulletin of Odessa State Academy of Civil Engineering and Architecture*, 83, 130-138. <https://doi.org/10.31650/2415-377X-2021-83-130-138>
 13. **Clemo T.** (2006). Flow in perforated pipes: a comparison of models and experiments. *SPE Production & Operations*, 21(2), 302-311. <https://doi.org/10.2118/89036-PA>
 14. **Kravchuk, A., Kochetov, G., & Kravchuk, O.** (2020). Pipelines designing for steady water collection along the path. *Problems of Water Supply, Sewerage and Hydraulic*, (33), 34-40. (*in Ukrainian*). <https://doi.org/10.32347/2524-0021.2020.33.34-40>
 15. **Naumenko, I. I., & Voloshchuk, V. A.** (2001). Mathematical models for hydraulic calculations of pipelines with discretely increasing flows. *Bulletin of RDTU*, 1(8), 88-99. (*in Ukrainian*).
 16. **Cho, J., Kim, C., Lim, K. J., Kim, J., Ji, B., & Yeon, J.** (2023). Web-based agricultural infrastructure digital twin system integrated with GIS and BIM concepts. *Computers and Electronics in Agriculture*, 215, 108441. <https://doi.org/10.1016/j.compag.2023.108441>
 17. **Rokochinskiy, A., Jeznach, J., Volk, P., Turcheniuk, V., Frolenkova, N., & Koptiuk, R.** (2019). Reclamation projects development improvement technology considering optimization of drained lands water regulation based on BIM. *Scientific Review Engineering and Environmental Sciences*, 28(3), 432-443. <https://doi.org/10.22630/PNIKS.2019.28.3.40>
 18. **Kravchuk, A., Cherniuk, V., Kochetov, G., Kravchuk, O., & Airapetian, T.** (2023). Determination of the particularities of the hydraulic friction factor variation of collecting drainage pipelines. *Eastern-European Journal of Enterprise Technologies*, 6(7(126)), 33-38. <https://doi.org/10.15587/1729-4061.2023.292258>
 19. **Kravchuk, A., & Kravchuk, O.** (2024). Determination of the required degree of surface perforation of collecting drainage pipelines. *Problems of Water Supply, Sewerage and Hydraulic*, 47, 29-34. (*in Ukrainian*). <https://doi.org/10.32347/2524-0021.2024.47.29-34>
 20. **Kravchuk, A., Kravchuk, O., Barladina, V., & Perebyinis, V.** (2023). Calculation of horizontal collective drainage pipelines of reclamation systems in the presence of a ground water level slope. *Girnichy, budivelni, dorozhni ta meliorativni mashini*, 102, 58-64. (*in Ukrainian*). <https://doi.org/10.32347/gbdmm.2023.102.0501>
 21. **Kravchuk, A., Kravchuk, O., Lomako, A., Kravchuk, O.** (2022). Variation of the collective drainage pipelines parameters when passing the transit flow. *Problems of Water supply, Sewerage and Hydraulics*, 41, 52-58. (*in Ukrainian*). <https://doi.org/10.32347/2524-0021.2022.41.52-58>
 22. **Kravchuk, A., Kravchuk, O.** (2025). Calculation of collecting drainage pipelines with variable cross-section. *Problems of Water supply, Sewerage and Hydraulics*, 52, 39-45. (*in Ukrainian*). <https://doi.org/10.32347/2524-0021.2025.52.39-45>

23. **Kravchuk, A. M., & Kravchuk, O. A.** (2020). Special issues of hydraulics of water supply and water sewerage systems: tutorial. Kyiv, Ukraine: KNUCA, 175. (in Ukrainian).

Визначення оптимальної середньої швидкості руху рідини в перерізах збірних дренажних трубопроводів змінного діаметра

Олександр Кравчук¹, Ольга Кравчук²

¹*Київський національний університет
будівництва і архітектури*

²*Національний транспортний університет*

Анотація. Аналіз умов роботи напірних збірних дренажних трубопроводів показав, що в якості останніх в основному розглядають трубопроводи постійного поперечного перерізу за своєю довжиною, які працюють зі змінною витратою. При цьому на початкових ділянках труб мають місце невеликі (менше допустимих) середні швидкості руху основного потоку, що часто призводить до осідання і накопичення в початкових перерізах значної кількості частинок навколишнього ґрунту. В кінцевих перерізах середня швидкість потоку набуває завищених значень, що спричиняє підвищення втрат напору. Таким чином, задача визначення оптимального значення середньої швидкості руху рідини в перерізах за довжиною збірних дренажних трубопроводів є актуальною.

Враховуючи викладене, метою даної роботи є розробка методики розрахунку величини оптимальної середньої швидкості руху рідини в перерізах за довжиною напірних збірних дренажних трубопроводів змінного поперечного перерізу (такого, що збільшується).

В даній роботі на основі безрозмірного аналізу системи диференціальних рівнянь, що описують рух рідини зі змінною витратою у збірних напірних дренажних трубопроводах змінного (такого, що збільшується) за довжиною перерізу, розроблена методика розрахунку оптимальної середньої швидкості руху рідини в перерізах трубопроводу. При аналізі в диференційному рівнянні змінної витрати другим членом, який враховує втрати напору, що пов'язані з впливом ефекту приєднання рідини вздовж шляху на загальні втрати напору у всьому трубопроводі, знехтувано, що практично не впливає на кінцеві результати розрахунків. При розгляді використано поняття умовного нескінченного довгого збірного трубопроводу або трубопроводу обмеженої довжини, що має нескінченну просякненість поверхні бічних стінок. Основними параметрами, які визначають характер зміни витрати вздовж збірних дренажних трубопроводів, прийнято коефіцієнт опору збірника $\zeta_{зб}$ і узагальнений параметр збірної дрени $A_{зб}$, який враховує її конструктивні і фільтраційні характеристики.

У результаті дослідження отримано просту та зручну методику визначення максимальної середньої швидкості у кінцевому перерізі напірного збірного дренажного трубопроводу, що враховує змінність діаметра та нерівномірність притоку за довжиною.

Ключові слова: збірний дренажний трубопровід, гідравлічний коефіцієнт тертя, коефіцієнт фільтрації, фільтраційний опір, змінна витрата.

Copyright (c) 2025, Authors. This is an open access article under the Creative Commons CC BY license