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Synthesis of optimal trajectories of a load with a jib crane

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Abstract. The article discusses the problem of synthesising optimal trajectories for moving load with a tower crane in order to increase its productivity and safety. A dynamic model of the system has been developed, including the boom, trolley and load suspended on a flexible rope, taking into account the pendulum oscillations of the load. To determine the laws of motion of the boom and trolley that ensure the specified load trajectory, the inverse kinematic problem has been solved. An optimisation approach has been proposed, aimed at minimising the duration of load transportation while complying with kinematic and dynamic constraints, in particular the maximum speeds of the trolley and boom rotation. To ensure smooth movement and reduce dynamic loads, regularisation methods are used to reduce the amplitude of load oscillations and eliminate unwanted changes in the direction of movement of the mechanisms. The numerical solution of the optimisation problem was performed using a modified particle swarm optimisation method (VCT-PSO), which demonstrated its effectiveness in selecting the optimal trajectory parameters. The influence of regularisation and the length of the flexible suspension on the dynamic characteristics of the system, in particular the amplitude of oscillations, peak forces and moments, as well as the duration of movement, was analysed. The results show that the use of regularisation significantly reduces dynamic loads and increases motion stability, although it increases transport time. Shortening the suspension length reduces motion time but can lead to an increase in oscillation amplitude. A new tool for analysing system dynamics in terms of deviations and velocity differences is proposed, which provides a visual assessment of the oscillation process and confirms the fulfilment of boundary conditions. The results obtained can be used to improve tower crane control systems and increase their efficiency on construction sites.

Keywords: tower crane, load trajectory, inverse kinematics, optimisation, pendulum dynamics, VCT-PSO, regularisation.

INTRODUCTION

The dynamic movement of loads by tower cranes involves complex interaction between the boom (a horizontal arm extending from the tower), the trolley (a platform moving along the boom), and the suspended load. This complexity arises primarily due to pendulum-like oscillations of the load. Optimising load movement trajectories is essential for reducing transport time, energy consumption, and wear on crane mechanisms, as well as for improving operational safety.

Recent research has focused on addressing these challenges using various control and optimisation strategies to enable efficient trajectory planning for tower cranes. According to [1-2], adaptive control systems incorporating fuzzy logic and vibration compensation have been proposed to handle system uncertainties. These approaches effectively suppress load oscillations, though they can be difficult to configure in practice.

Studies [3-4] investigate real-time sensor-based vibration damping systems. These methods reduce dynamic loads on crane components, but their effectiveness heavily depends on sensor precision and reliability, which can be problematic on construction sites.

The impact of suspension cable length is examined in [5], where it is shown that shorter cables improve load stabilisation time but also increase vibration amplitude, highlighting the need for a trade-off between velocity and smoothness.

In [6], regularisation is introduced as a technique to smooth trajectory profiles. While this approach reduces oscillations and mechanical stress, it increases total movement time, making it suitable when stability and equipment longevity are priorities.

In study [7], a method of trajectory control of a crane using parametric optimisation based on polynomials was proposed. The authors implemented control of the crane trolley movement, taking into account the variable length of the rope. This approach made it possible to significantly reduce deviations from the trajectory. The disadvantage is the need for precise parameter tuning and high sensitivity to model errors.

In [8], a model of a gantry crane with a flexible suspension, described by partial differential equations, is considered. The developed control ensures effective damping of load oscillations. The method has been experimentally verified on a laboratory setup, but the complexity of the model limits its use for online controllers.

The authors [9] investigate the planning of the motion trajectories of a tower crane trolley with a double pendulum suspension. Time polynomial optimisation is applied for each motion segment. The work is valuable in that it takes into account the complex dynamics of the system, but does not contain an analysis of the influence of external disturbances.

PURPOSE OF THE PAPER

The aim of the work is to increase the productivity of tower cranes by synthesising optimal load movement trajectories.

RESEARCH OBJECTIVES

1. Develop a mathematical model of a tower crane system, taking into account the boom, trolley, and suspended load.
2. Solve the inverse kinematics problem to determine the laws of motion that implement a predetermined load trajectory.
3. State and solve an optimisation problem aimed at minimising transport time, taking into account kinematic and dynamic constraints.
4. Analyse the effect of regularisation and suspension length on load oscillation and system performance.

RESEARCH RESULTS

Solving the problem of controlling the movement of a tower crane load requires a detailed analysis of the dynamic model of the system (Fig. 1). The study concerns a system consisting of three main elements: a boom, a trolley, and a load suspended on a flexible rope. The main objective of the study is to determine the optimal trajectory of the load and to synthesize the optimal laws for controlling the movement of the boom and trolley, which ensure the optimal trajectory.

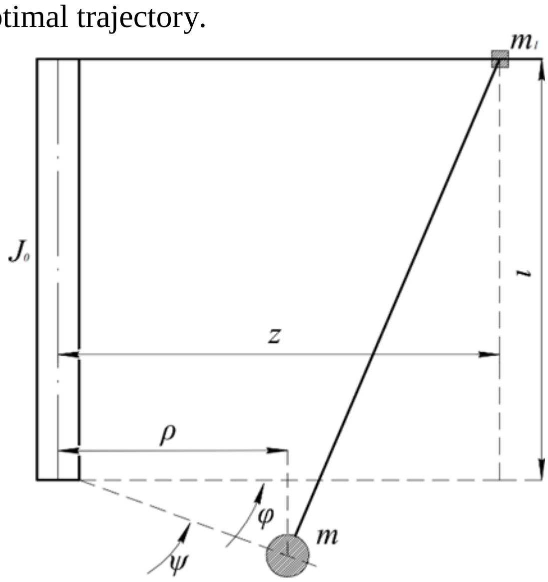

Fig. 1. Dynamic model of load movement by a tower crane

The following notations were used to develop the mathematical model of the system (Fig. 1): z is the coordinate of the trolley moving along the boom; ρ is the coordinate of the load along the boom axis; φ is the angular coordinate of the boom rotation relative to the vertical axis; ψ is the angular coordinate of the pendulum oscillations of the load in the direction perpendicular to the plane of the boom. Thus, the system included four independent coordinates z, ρ, φ, ψ, which determine the position of the trolley, the crane's rotation, and the pendulum oscillations of the load.

A mathematical model is represented by a system of equations that determine the behaviour of structural elements [10]:

$$\begin{cases} m_1\ddot{z} - m_1 z\dot{\varphi}^2 = F - \dfrac{mg}{l}\left(z - \rho\left(1 - \dfrac{(\varphi-\psi)^2}{2}\right) - W\,\mathrm{sgn}(\dot{z})\right); \\ \ddot{\rho} - \rho\dot{\psi}^2 = -\dfrac{g}{l}\left(\rho - z\left(1 - \dfrac{(\varphi-\psi)^2}{2}\right)\right); \\ \left(J_0 + m_1 z^2\right)\ddot{\varphi} + 2m_1 z\dot{z}\varphi = M - \dfrac{mg}{l}\rho z(\varphi-\psi) - M\,\mathrm{sgn}(\dot{\varphi}); \\ \rho\ddot{\psi} + 2\dot{\rho}\dot{\psi} = \dfrac{g}{l} z(\varphi-\psi), \end{cases} \tag{1}$$

where l is the length of the flexible suspension on which the load is fixed; m is the mass of the load; J is the total moment of inertia of the tower relative to its rotation axis.

Further calculations will be based on only two equations of the system (1), which describe the movement of the mass of the load m:

$$\begin{cases} \ddot{\rho} - \rho\dot{\psi}^2 = -\dfrac{g}{l}\left(\rho - z\left(1 - \dfrac{(\varphi-\psi)^2}{2}\right)\right); \\ \rho\ddot{\psi} + 2\dot{\rho}\dot{\psi} = \dfrac{g}{l} z(\varphi-\psi). \end{cases} \tag{1a}$$

If the trajectory of the load is known (the laws of change in the angular position of the boom ψ and the position of the trolley ρ are known), then the problem is to determine the laws of change in z and φ, which would ensure the implementation of the given trajectory of the load. Thus, it is necessary to determine how to control the movement of the boom and trolley so that the load moves according to the given trajectory.

Dividing the second equation of the system (1a) by $\dfrac{l}{g}$, we obtain:

$$\frac{l}{g}\left(\rho\ddot{\psi} + 2\dot{\rho}\dot{\psi}\right) = z(\varphi-\psi), \tag{2}$$

After entering the designations:

$$\frac{l}{g}\left(\rho\ddot{\psi} + 2\dot{\rho}\dot{\psi}\right) = R, \tag{3}$$

we obtain the expression that follows from the second equation of system (1a):

$$R = z(\varphi-\psi). \tag{4}$$

We have:

$$\frac{R}{z} = \varphi - \psi, \tag{5}$$

or

$$\varphi = \frac{R}{z} + \psi. \tag{6}$$

So, knowing the law of change of the coordinate $z(t)$, we can use equation (6) to find the desired law $\varphi(t)$. At this stage, it is necessary to find the expression $\varphi=\varphi(\psi(t), \rho(t))$. Taking into account expression (5), we rewrite the first equation of system (1a) in the following form:

$$\ddot{\rho} - \rho\dot{\psi}^2 = -\frac{g}{l}\left(\rho - z + z\frac{\left(\frac{R}{z}\right)^2}{2}\right). \tag{7}$$

Dividing the left and right sides of equation (7) by $-g/l$, we obtain:

$$-\frac{l}{g}\left(\ddot{\rho} - \rho\dot{\psi}^2\right) = \rho - z + \frac{R^2}{2z}. \tag{8}$$

Let's introduce another notation:

$$\frac{l}{g}\left(\rho\dot{\psi} - \ddot{\rho}\right) = K. \tag{9}$$

Then, taking into account equation (8), we obtain:

$$K = \rho - z + \frac{R^2}{2z} = \frac{2z\rho - 2z^2 + R^2}{2z}. \tag{10}$$

To simplify equation (10), we will represent it as a quadratic algebraic equation:

$$z^2 + z(K-\rho) - \frac{R^2}{2} = 0. \quad (11)$$

By solving the resulting quadratic equation (11), we obtain two roots:

$$z_{1,2} = \frac{1}{2}\left(-K \pm \sqrt{2R^2 + (K-\rho)^2 + \rho}\right). \quad (12)$$

It is necessary to determine which of them was acceptable and can be used in further calculations. To do this, each solution should be analysed in terms of its compliance with the actual dynamics of the system. The analysis was performed on the basis of plots. In order to build them, the problem was solved, which consisted of moving the load from the initial point with coordinates (x_0, y_0) to the final point (x_T, y_T). In the case under consideration, the initial coordinates were (x_0, y_0)=(10, 10), and the final coordinates were (x_T, y_T)=(2, 1). To ensure such a move, the corresponding values of the coordinates z and φ were calculated for both cases of the roots (12).

To build these plots, the initial parameters of the load movement were set, namely the coordinates ρ and ψ, which determined its trajectory. They were chosen in such a way as to ensure that the initial and final points of the load movement were connected in accordance with the boundary conditions. To determine the movement parameters, it was also assumed that the length of the flexible suspension was 5 m.

The laws ψ and ρ were obtained, which corresponded to the initial (x_0, y_0) and final (x_T, y_T) coordinates of the load movement and satisfied the boundary conditions of movement, i.e. the conditions of the beginning and end of movement from the resting state:

$$\psi = (T^7\psi_0 + 20t^7(\psi_0 - \psi_T) + 84t^5T^2 \times \times(\psi_0 - \psi_T) + 70t^6T(-\psi_0 + \psi_T) + 35t^4 \times \times T^3(-\psi_0 + \psi_T))T^{-7} \quad (13)$$

where ψ_0 and ψ_T are the angular initial and final positions of the load:

$$\psi_0 = arctg\left(\frac{y_0}{x_0}\right); \quad (14)$$

$$\psi_T = arctg\left(\frac{y_T}{x_T}\right); \quad (15)$$

$$\rho = (T^7\rho_0 + 20t^7(\rho_0 - \rho_T) + 84t^5T^2 \times (\rho_0 - \rho_T) + 70t^6T(-\rho_0 + \rho_T) + 35t^4 \times \times T^3(-\rho_0 + \rho_T))T^{-7} \quad (16)$$

where ρ_0 and ρ_T are the linear initial and final positions of the load:

$$\rho_0 = \sqrt{x_0^2 + y_0^2}; \quad (17)$$

$$\rho_T = \sqrt{x_T^2 + y_T^2}. \quad (18)$$

Taking into account expressions (13)-(18), as well as those obtained earlier (12), plots were built (Fig. 2). In Fig. 2, the black curve corresponds to the root of z_1, and the grey curve to the root of z_2.

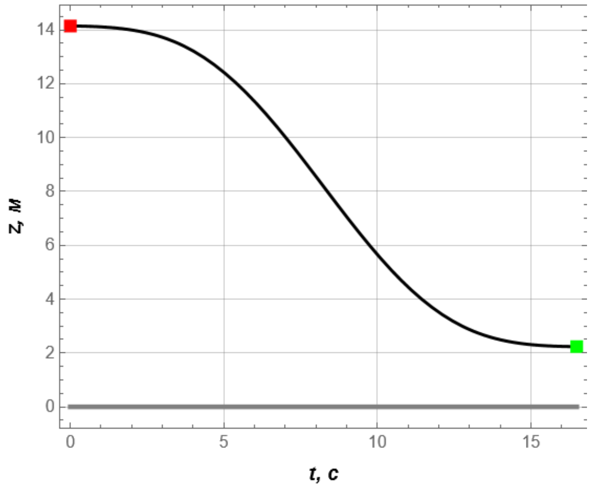

Fig. 2. Plots for two variants of the roots (12)

The analysis of these plots showed that the second root z_2 does not provide the boundary conditions for the trolley's movement. The black curve, which corresponded to the root z_1, provided a correct connection between the points of the initial (red dot) and final positions (green dot). Thus, based on the analysis of plots, it was decided to use the root z_1 in further calculations.

The inverse kinematics problem was solved with consideration of the pendulum effects of the load movement. The next stage of the study is to determine the law of motion of the load and build its trajectory. This, in turn, made it possible to determine the law of motion of the suspension point. To determine the law of motion of the load and its trajectory, it was necessary to set the initial conditions that determined from which state the motion began and in which it ended. Establishing these conditions ensured correct modelling of the load movement and allowed further calculations of the system's motion dynamics.

The initial conditions of the system movement in the absence of initial pendulum oscillations of the load and the resting state of the trolley and boom are set as follows:

$$\begin{cases} z(0) = \rho(0) = \rho_0; \\ \varphi(0) = \psi(0) = \psi_0; \\ \dot{z}(0) = \dot{\rho}(0) = 0; \\ \dot{\varphi}(0) = \dot{\psi}(0) = 0. \end{cases} \qquad (19)$$

The initial polar coordinates of the load expressed in terms of Cartesian coordinates are represented as expressions (14), (17).

The final motion conditions can be written in the same way as the initial ones:

$$\begin{cases} z(T) = \rho(T) = \rho_T; \\ \varphi(T) = \psi(T) = \psi_T; \\ \dot{z}(T) = \dot{\rho}(T) = 0; \\ \dot{\varphi}(T) = \dot{\psi}(T) = 0. \end{cases} . \qquad (20)$$

The end conditions for the angular coordinates are also written in the same way. The angles φ and ψ had to be equal to each other, which corresponded to the condition that the load would reach its final position. In addition, the final velocities of all structural elements were equal to zero, which ensured that the system would stop at a given point. Thus, the initial and final conditions of motion correspond to the resting state of the system elements. In the future, it is necessary to obtain the boundary conditions for the movement of the load (functions ρ and ψ), which provide conditions (19), (20) and are expressed through the boundary conditions for the movement of the trolley and boom.

Let us first consider the initial conditions. Taking into account the dependencies (4), (6), (10) and (12), we can write down the initial conditions for the movement of the trolley and boom, expressed through the initial conditions for the movement of the load:

$$z(0) = \frac{1}{2}\left(\frac{\ddot{\rho}_0 l}{g} + \rho_0 + \sqrt{\frac{2\ddot{\psi}_0^2 l^2 \rho_0^2 + \left(\ddot{\rho}_0 l + g\rho_0\right)^2}{g^2}} \right) \qquad (21)$$

$$\dot{z}(0) = \frac{\dddot{\rho}_0 g l + \dfrac{l\left(\dddot{\rho}_0 \ddot{\rho}_0 l + \dddot{\rho}_0 g\rho_0 + 2\dddot{\psi}_0 \ddot{\psi}_0 l \rho_0^2\right)}{\sqrt{\dfrac{2\ddot{\psi}_0^2 l^2 \rho_0^2 + \left(\ddot{\rho}_0 l + g\rho_0\right)^2}{g^2}}}}{2g^2} \qquad (22)$$

$$\varphi(0) = \frac{2\ddot{\psi}_0 l \rho_0}{\ddot{\rho}_0 l + g\left(\rho_0 + \sqrt{\dfrac{\ddot{\rho}_0^2 l^2 + 2\ddot{\rho}_0 g l \rho_0 + \left(g^2 + 2\ddot{\psi}_0^2 l^2\right)\rho_0^2}{g^2}} \right)} + \psi_0 \qquad (23)$$

$$\dot{\varphi}(0) = \frac{2l\rho_0\left(\dddot{\psi}_0 \ddot{\rho}_0 l - \dddot{\rho}_0 \ddot{\psi}_0 l - \dddot{\rho}_0 \ddot{\psi}_0 l + \dddot{\psi}_0 g \rho_0\right)}{g\sqrt{\dfrac{2\ddot{\psi}_0^2 l^2 \rho_0^2 + \left(\ddot{\rho}_0 l + g\rho_0\right)^2}{g^2}}\left(\ddot{\rho}_0 + g\rho_0 + g\sqrt{\dfrac{2\ddot{\psi}_0^2 l^2 \rho_0^2 + \left(\ddot{\rho}_0 l + g\rho_0\right)^2}{g^2}} \right)} \qquad (24)$$

Taking into account these records, we obtained a system of equations that meets the initial conditions (19) and in a reduced (compact) form has the following form:

$$\begin{cases} z(0) - \rho(0) = 0; \\ \varphi(0) - \psi(0) = 0; \\ \dot{z}(0) = 0; \\ \dot{\varphi}(0) = 0; \end{cases} . \qquad (25)$$

The solution to system (25) is as follows:

$$\ddot{\psi}(0) = \dddot{\psi}(0) = \ddot{\rho}(0) = \dddot{\rho}(0) = 0 . \qquad (26)$$

Similar calculations for the final conditions will be given:

$$\ddot{\psi}(T) = \dddot{\psi}(T) = \ddot{\rho}(T) = \dddot{\rho}(T) = 0 . \qquad (27)$$

So we can write down the final version of the boundary conditions for the movement of the load:

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$$\begin{cases} \psi(0) = \psi_0, \ \dot{\psi}(0) = \ddot{\psi}(0) = \dddot{\psi}(0) = 0; \\ \rho(0) = \rho_0, \ \dot{\rho}(0) = \ddot{\rho}(0) = \dddot{\rho}(0) = 0; \\ \psi(T) = \psi_T, \dot{\psi}(T) = \ddot{\psi}(T) = \dddot{\psi}(T) = 0; \\ \rho(T) = \rho_T, \ \dot{\rho}(T) = \ddot{\rho}(T) = \dddot{\rho}(T) = 0. \end{cases} \quad (28)$$

These conditions will be used to calculate the optimal trajectories for load movement.

Let's formulate the optimisation problem. It includes four components: 1) a mathematical model of the system (1a); 2) motion boundary conditions (19), which will be used in the form of (28) in further calculations; 3) optimisation criterion; 4) constraints.

Since the first two points of the problem statement have already been presented in the previous para plots, we will focus on the third and fourth. The optimisation criterion for this problem is the duration of the load movement:

$$\int_0^T dt = T \rightarrow \min. \quad (29)$$

The use of criterion (29) makes it possible to increase the productivity of load handling at the construction site.

The problem introduces constraints on the maximum linear and angular velocities of the trolley and boom rotation, respectively:

$$\begin{aligned} &0 \le \dot{z}(t) \le \dot{z}_{\max}; \\ &0 \le \dot{\varphi}(t) \le \dot{\varphi}_{\max}, \end{aligned} \quad (30)$$

where $\dot{z}_{\max}$ and $\dot{\varphi}_{\max}$ − are the maximum acceptable values of the trolley and boom velocities, respectively ($\dot{z}_{\max}$=0.8 m/s and $\dot{\varphi}_{\max}$ =0.105 rad/s). The constraints on the maximum trolley velocities $\dot{z}_{\max}$=0.8 m/s and boom slew rates $\dot{\varphi}_{\max}$=0.105 rad/s are based on the typical technical characteristics of modern tower cranes.

To solve the problem (1a), (28)-(30), we reduced it to an unconstraints optimisation problem. To do this, the functions on which the solution to the problem is sought were represented in the form of a superposition:

$$\begin{aligned} &\psi = \psi_1 + \psi_2; \\ &\rho = \rho_1 + \rho_2, \end{aligned} \quad (31)$$

where ψ_1 and ρ_1 are functions that ensure the fulfilment of boundary conditions (28); ψ_2 and ρ_2 are polynomial functions that ensure the fulfilment of zero boundary conditions up to and including the third derivative and contain free parameters (coefficients) that will be used to minimise the criterion (29):

$$\psi_2 = \left(\frac{t}{T}\right)^4 \left(\frac{T-t}{T}\right)^4 \times$$
$$\times \left(A_0 + A_1\left(\frac{t}{T}\right) + A_2\left(\frac{t}{T}\right)^2 + A_3\left(\frac{t}{T}\right)^3 \right)(\psi_T - \psi_0); \quad (32)$$

$$\rho_2 = \left(\frac{t}{T}\right)^4 \left(\frac{T-t}{T}\right)^4 \times$$
$$\times \left(B_0 + B_1\left(\frac{t}{T}\right) + B_2\left(\frac{t}{T}\right)^2 + B_3\left(\frac{t}{T}\right)^3 \right)(\rho_T - \rho_0) \quad (33)$$

The coefficients in expressions (32), (33) $\left(\frac{t}{T}\right)^4$ and $\left(\frac{T-t}{T}\right)^4$ provide zero boundary conditions for motion. Thus, functions (32), (33) do not affect the given boundary conditions of problem (28).

To ensure that the constraints (30) are met, the following penalty functions were constructed:

$$P_{\dot{\varphi}} = \delta_{\dot{\varphi}} \begin{cases} 0, \ if \quad 0 \le \dot{\varphi}(t) \le \dot{\varphi}_{\max}; \\ \sqrt{(\dot{\varphi}_{\max} - \dot{\varphi}(t))^2}, \ if \quad \dot{\varphi}(t) > \dot{\varphi}_{\max}; \\ \sqrt{(\dot{\varphi}(t))^2}, \ if \quad \dot{\varphi}(t) < 0; \end{cases}$$

$$P_{\dot{z}} = \delta_{\dot{z}} \begin{cases} 0, \ if \quad 0 \le \dot{z}(t) \le \dot{z}_{\max}; \\ \sqrt{(\dot{z}_{\max} - \dot{z}(t))^2}, \ if \quad \dot{z}(t) > \dot{z}_{\max}; \\ \sqrt{(\dot{z}(t))^2}, \ if \quad \dot{z}(t) < 0, \end{cases} \quad (34)$$

where $\delta_{\dot{\varphi}}$ and $\delta_{\dot{z}}$ − are weighting factors that reduce the dimensionality of the penalty functions to the dimensionality of the criterion (29) and take into account the importance of meeting the constraints (30).

During the calculations, the trolley velocity and the boom velocity were determined, and then their maximum values were determined. If the maximum linear velocity of the trolley exceeded the maximum value of 0.8 m/s, this meant that the acceptable limit was exceeded.

In this case, the penalty function $P_{\dot{\varphi}}$ became large.

The penalty function $P_{\dot{z}}$ worked in a similar way. In the case when the constraints (30) were met, both penalty functions were equal to zero. The use of penalty components made it possible to provide constraints on the maximum velocities of the boom and trolley.

Thus, the problem of synthesising an optimal trajectory was presented in the form of an unconstraint optimisation problem:

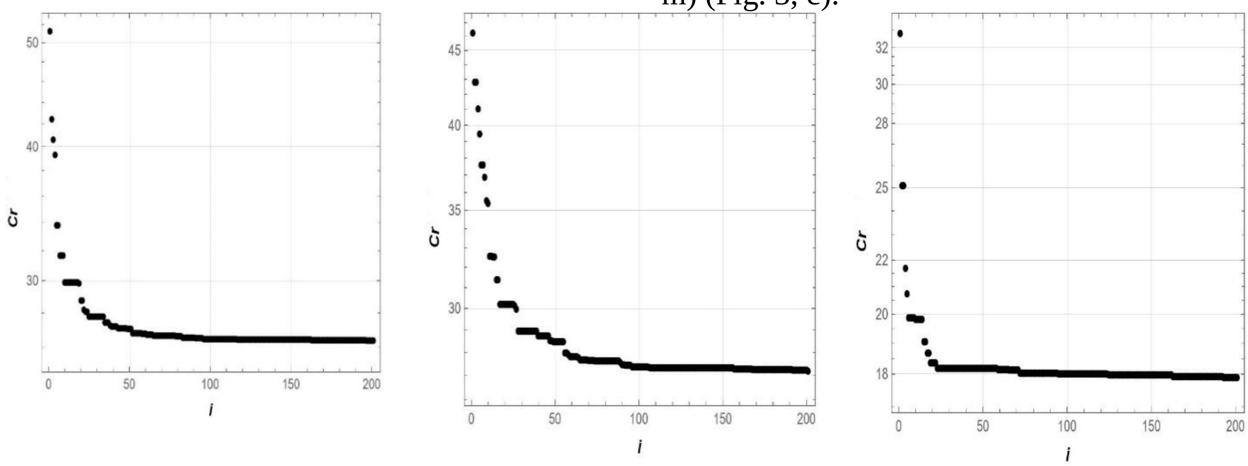
$$\begin{aligned} &Cr(A_0, A_1, A_2, A_3, B_0, B_1, B_2, B_3, T) = \\ &= T + P_{\dot{\varphi}} + P_{\dot{z}} + \\ &+ \lambda \left\| \left(A_0, A_1, A_2, A_3, B_0, B_1, B_2, B_3 \right) \right\| \to \min, \end{aligned} \tag{35}$$

where λ − is a regularisation factor that ensures the importance of reducing the coefficients A_0, A_1, A_2, A_3 and B_0, B_1, B_2, B_3. The introduction of the regularisation component in the objective function was due to the fact that it allows finding solutions to problems (1a), (28)-(30) with insignificant values of the coefficients A_0, A_1, A_2, A_3 and B_0, B_1, B_2, B_3. This will reduce the fluctuation of the solution, which will have a positive impact on the practical implementation of the results obtained.

Thus, the problem is to determine the optimal values of the corresponding coefficients A_0, A_1, A_2, A_3 and B_0, B_1, B_2, B_3, which determine the shape of the trajectory and the duration of the movement T.

The VCT-PSO numerical optimisation method [11] was used to solve the optimisation problem. The number of iterations was 200, and the dimension of the swarm was 20. The search arguments varied from -100 to 100. As a result of the optimisation procedure, it was possible to find the values of the parameters that ensured the minimum time for moving the load from point A to point B while meeting all the conditions of the problem. The optimal travel time was determined in accordance with the found values of the control coefficients.

Fig. 3 shows the plots of convergence of the objective function (35) when applying the VCT-PSO method for different variants of solving the problem. The plots show the dependence of the objective function convergence on the iteration number for three calculation options: 1) without regularisation (λ=0; l=5 m) (Fig. 3, a); 2) with regularisation (λ=0.1; l=5 m) (Fig. 3, b); 3) with regularisation and a changed length of the flexible suspension (λ=0.1; l=2.5 m) (Fig. 3, c).



a *b* *c*

Fig. 3. Plots of convergence of the objective function (35) for the cases of calculations: a) the first; b) the second; c) the third

For the first case, a stable value of the objective function is quickly reached after about the 40th iteration. The initial fluctuations in the function values are small and disappear quite quickly. For the second variant, the optimised value of the objective function is higher than for the first. In addition, the algorithm's convergence process is somewhat slower. This indicates that regularisation increases the topological complexity of the objective function.

For the third calculation variant, reducing the length of the flexible suspension practically does not change the nature of the algorithm's convergence compared to the second variant, but slightly reduces the achieved value of the objective function.

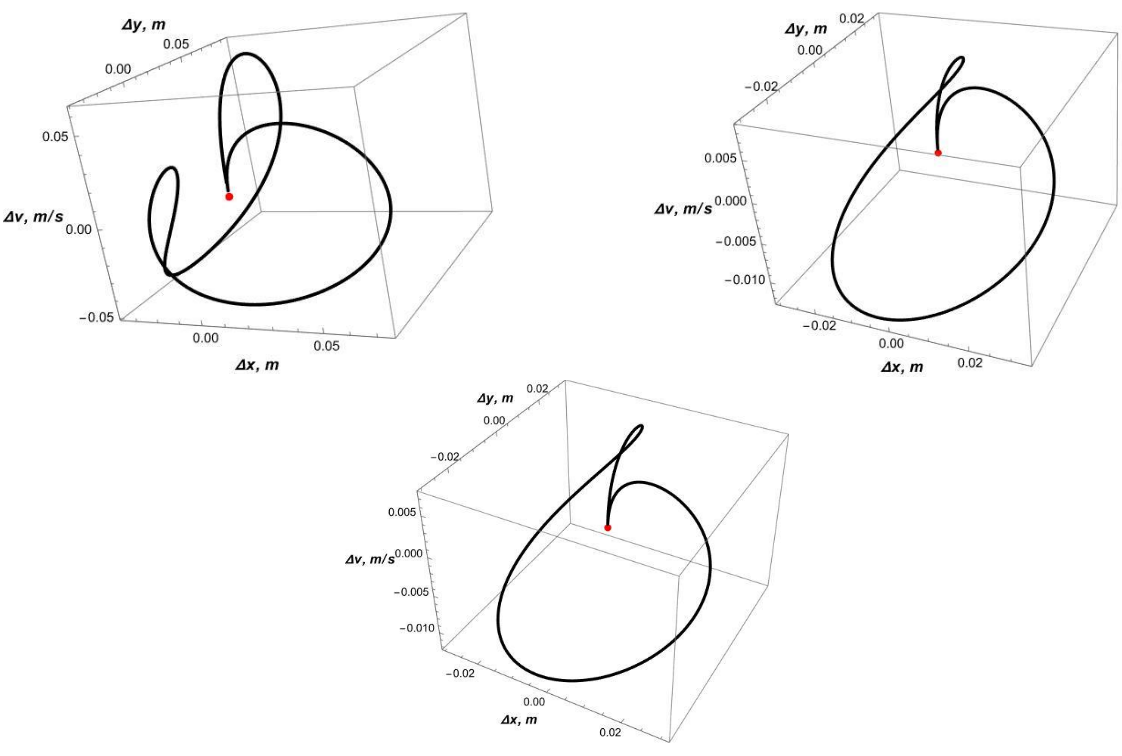
This indicates that a shorter length of the flexible suspension allows achieving a better optimal value of the objective function.

In addition, Table I shows the numerical values of the obtained coefficients A_0, A_1, A_2, A_3, B_0, B_1, B_2, B_3 and the duration of the movement T.

All plots are presented in three variants of calculations, which made it possible to compare the results and analyse each of the variants of the problem solved (Fig. 4). In Fig. 4, the left column of the plots corresponds to the first variant of the problem, the right column - to the second, and the lowest column - to the third.

Table I. Coefficients A_0, A_1, A_2, A_3, B_0, B_1, B_2, B_3 and movement duration T for the solved problems

Numerical values of the objective function arguments	Problem parameters		
	λ=0; l=5 m	λ=0.1; l=5 m	λ=0.1; l=2.5 m
A_0	62.4987	8.52567	0.11311
A_1	-61.4522	-1.1071	-2.06183
A_2	-98.4453	-5.41954	-1.34649
A_3	-99.124	-4.3217	-1.31595
B_0	98.4241	-8.89722	-0.0472256
B_1	-100	-3.6694	-0.021498
B_2	10.4286	-1.27586	0.217561
B_3	43.9062	-1.32075	-0.327916
T	17.8862	25.7081	26.1023



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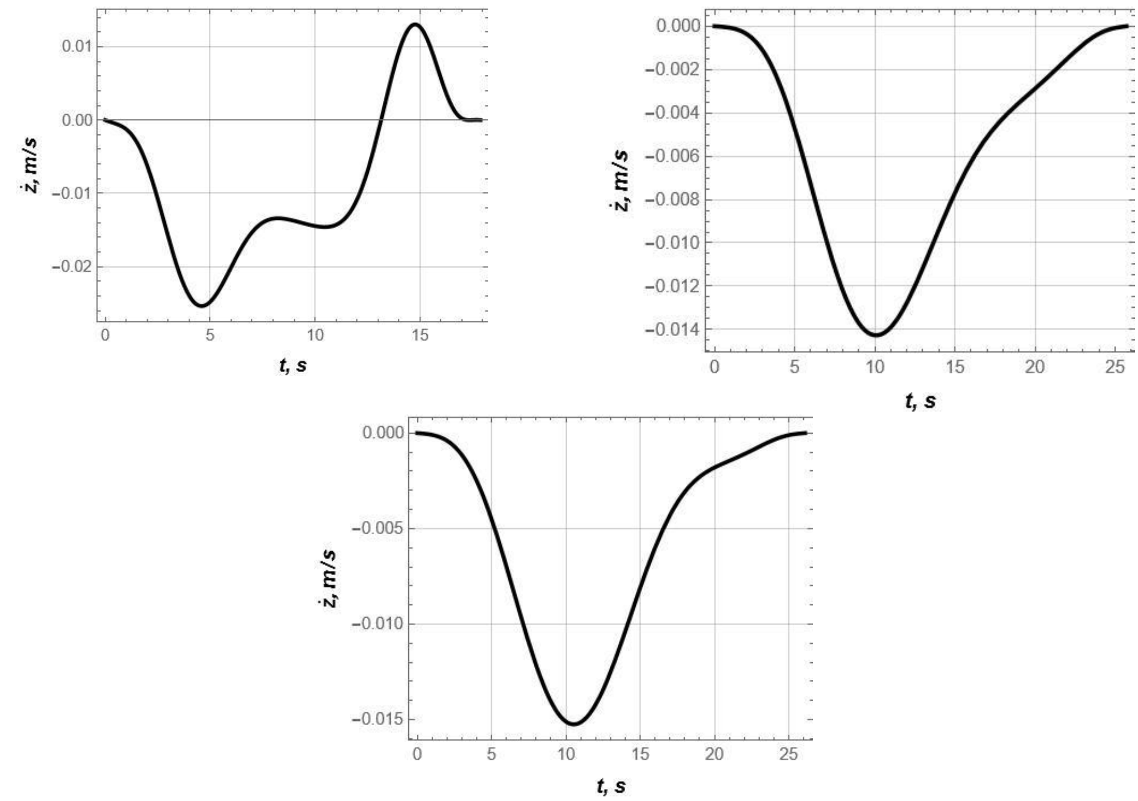
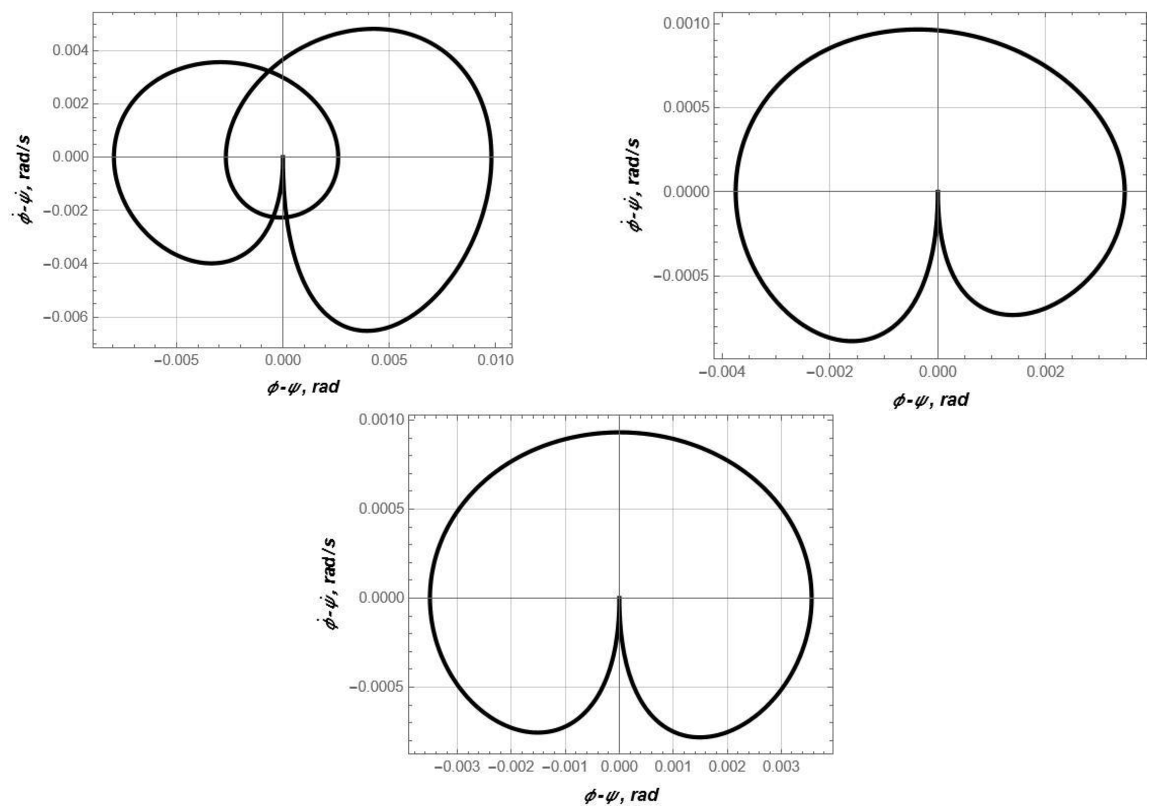

0.01
0.00
−0.01
−0.02
ż, m/s
0
5
10
15
t, s
0.000
−0.002
−0.004
−0.006
−0.008
−0.010
−0.012
−0.014
ż, m/s
0
5
10
15
20
25
t, s
0.000
−0.005
−0.010
−0.015
ż, m/s
0
5
10
15
20
25
t, s

b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0.002
0.000
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−0.006
φ̇-ψ̇, rad/s
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0.010
φ-ψ, rad
0.0010
0.0005
0.0000
−0.0005
φ̇-ψ̇, rad/s
−0.004
−0.002
0.000
0.002
φ-ψ, rad
0.0010
0.0005
0.0000
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φ̇-ψ̇, rad/s
−0.003
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0.002
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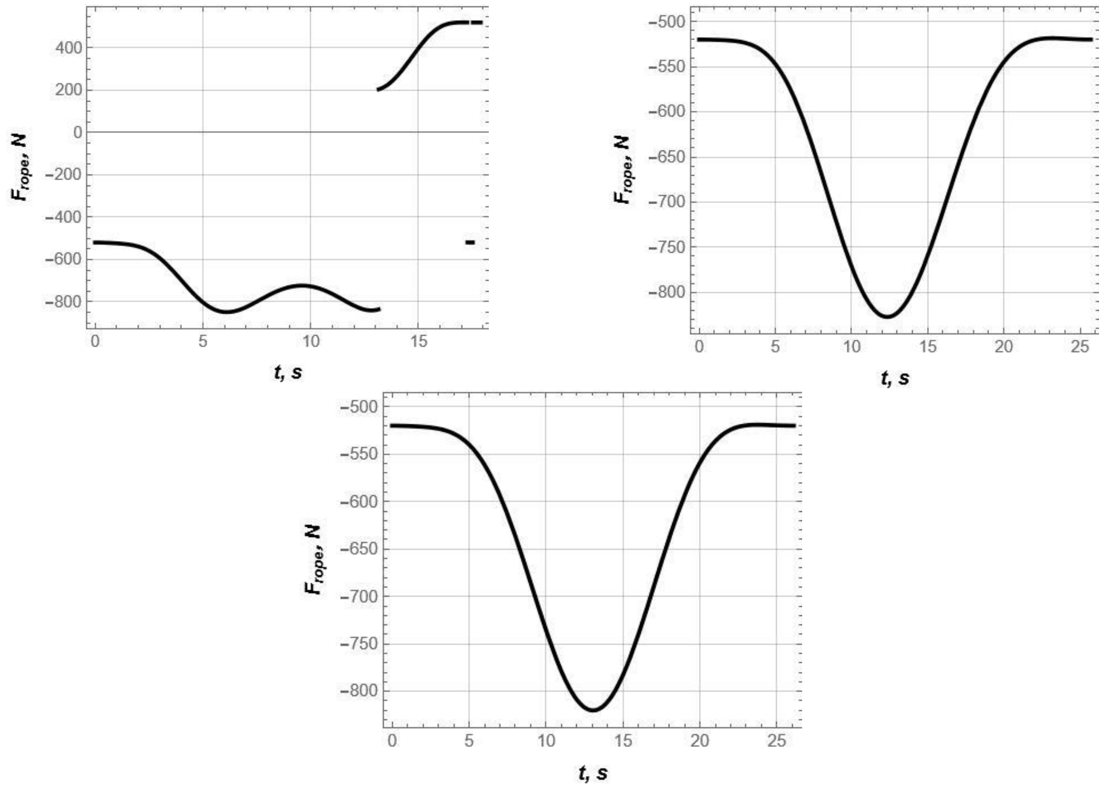
φ-ψ, rad

c



d

Fig. 4. Plots of the characteristics of the movement of tower crane mechanisms and load according to optimal laws: a) non-classical phase trajectory of load movement b) trolley movement velocity; c) phase trajectory of the pendulum oscillations of the load in the plane of movement of the trolley; d) total driving force of the drive of the trolley movement mechanism

From Fig. 4, it can be seen that the introduction of the regularisation component $\lambda = 0.1$ in the structure of the objective function (35) allowed to significantly reduce the amplitude of the load oscillations. The value of Δx decreased from about 0.20 m to 0.04 m. The amplitude Δy decreased from 0.10 m to 0.04 m, and the velocity Δv decreased from 0.20 m/s to 0.02 m/s. In addition, the regularisation completely eliminated repeated oscillation cycles, ensuring a smooth decay of the trajectory in one cycle. Reducing the length of the flexible suspension from 5 m to 2.5 m during regularisation slightly increased the maximum deviations (Δx increased to 0.10 m and Δv to 0.06 m/s), but at the same time significantly accelerated the damping of the oscillatory movement of the load.

The described tool for analysing the oscillatory motion of a pendulum system is new; it allows analysing the motion in the Δx, Δy and Δv coordinates, which reflect the deviation of the load from the suspension point in position (along the *x* and *y* axes) and the difference between the absolute velocities of the suspension point and the load itself. The use of this tool made it possible to obtain complete and clear information about the system's dynamic behaviour. In particular, it became possible not only to qualitatively assess the nature of the load's oscillations, but also to quantitatively trace their amplitude and decay rate. Visualisation of the movement in the selected coordinates made it possible to clearly determine whether the oscillatory process is completed before the system stops completely. The graphical method (Fig. 3, a) clearly illustrates the pendulum behaviour of the load and simplifies dynamic analysis.

From Fig. 4, b) shows that the introduction of regularisation for a 5 m long suspension made it possible to reduce the amplitude of the trolley's velocity from the range of -0.033...+0.027 m/s to -0.014...+0.003 m/s, and also reduced the oscillation damping time by half. Reducing the length of the suspension to

2.5 m slightly increased the amplitude to -0.019...+0.002 m/s, but at the same time accelerated the decay process even more (by 30 %). From Fig. 4 c) shows that with the use of the regularisation component, the amplitude of the load's linear deflection drops from 0.04 to 0.01 m, the velocity from 0.03 to 0.005 m/s, and the oscillation damping time from 12 to 6 s. With a shorter length of the flexible suspension (l = 2.5 m), the amplitude of the load oscillations increases to 0.015 m, the velocity - to 0.008 m/s, but the damping time is reduced to 4.2 s (by 30%). From Fig. 4, d) shows that the use of the regularisation component reduces the force from 850 to 800 N (5.88%) and the force curve profile becomes single-peaked. At l = 2.5 m, the force increases to 820 N (2.5%), and the damping time decreases to 4.8 s (20%).

Thus, it can be said that the use of the regularisation component significantly reduces the amplitude of load oscillations by 75-84%, eliminates additional oscillation cycles and reduces the decay time by 50% (from 10-12 s to 5-6 s). The peak driving torques are reduced by 46.15% (from 65 to 35 kN-m) and the force by 5.88% (from 850 to 800 N), which ensures stable movement. Reducing the length of the suspension to 2.5 m increases the amplitude of oscillations by 50-60%, but reduces the oscillation damping time by 20-30% (from 5-6 s to 3.5-4.8 s) and shortens the oscillation period. The peak driving torque decreases by 5.71% (to 33 kN-m), while the force increases by 2.5% (to 820 N).

For a more complete analysis of the results obtained, we calculated separate evaluation indicators for the three cases under consideration (Table II). The analysis of the data in Table II allows us to establish the following patterns of influence of the regularisation parameter λ on the qualitative and quantitative indicators of tower crane movement: 1) the duration of the movement increased by 43.7% (from 17.89 s to 25.71 s); 2) the maximum acceleration of the trolley and boom significantly decreased by 77.3% and 56.5%, respectively; 3) the maximum driving force in the rope decreased by 2.55% (from 848.95 N to 827.33 N); 4) the maximum driving torque of the slewing mechanism decreased by 50.2% (from 72676.9 to 36201.5 N-m); 5) the maximum power of the trolley movement mechanism decreased by 43.5% (from 19.93 to 11.26 W) and the maximum power of the crane slewing mechanism decreased by 46.8% (from 5081.6 to 2705.5 W). In addition, the rms values of the indicators (acceleration, deflection, driving forces, moments, power) have also significantly decreased, indicating smoother movement and reduced dynamic loads.

Thus, although the introduction of regularisation increases the movement duration by approximately 44%, it provides a significant reduction in dynamic loads, driving forces and power consumption of the drives, which reduces equipment wear and avoids unnecessary changes in the direction of movement of the electric drive.

The effect of reducing the length of the flexible suspension (from l=5 m to l=2.5 m) was analysed and the following patterns were established: 1) the duration of the movement decreased by 35.8% (from 25.71 to 16.50 s); 2) the maximum acceleration of the trolley increased significantly (from 0.0027 to 0.2968 m/s²); 3) the acceleration of the boom decreased by 39.9% (from 0.0145 to 0.0087 m/s²); 4) the maximum load deflection in the plane of the boom increased 5.7 times (from 0.0296 to 0.1696 m); 5) the maximum driving force in the rope of the trolley movement mechanism was significantly reduced by 27% (from 827.33 to 603.70 N); 6) the maximum driving torque of the crane slewing mechanism was reduced by another 28.8% (from 36201.5 to 25758.2 N-m).

Thus, it can be said that the introduction of regularisation (λ=0.1) is recommended, as it reduces dynamic loads, wear and tear on mechanisms, and eliminates the need for frequent reversal of the trolley and boom drive

CONCLUSIONS

1) As a result of solving the inverse kinematics problem, the laws of change in the coordinates z (position of the trolley) and φ (angle of rotation of the boom) were determined, which ensure the implementation of a given trajectory of load movement, taking into account its pendulum movement.

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Table II. Estimated indicators of movement of tower crane mechanisms and load according to optimal laws

Evaluation indicator	Units	Problem parameters		
		λ=0; l=5 m	λ=0.1; l=5 m	λ=0.1; l=2.5 m
1	2	3	4	5
Maximum acceleration of the trolley	m/s²	0.0119067	0.00269883	0.296796
Maximum boom acceleration	rad/s²	0.0332584	0.0144578	0.0086886
Maximum load deflection in the boom plane	m	0.0318426	0.0296264	0.169644
Maximum load deflection in the plane perpendicular to the boom plane	rad	0.0098043	0.00375357	0.0122082
Maximum value of the driving force in the rope of the trolley movement mechanism	N	848.953	827.329	603.697
Maximum value of the driving torque of the crane slewing mechanism	N·m	72676.9	36201.5	25758.2
Maximum power value of the trolley movement mechanism	W	19.925	11.2623	655.725
The maximum power value of the crane slewing mechanism	W	5081.6	2705.48	715.696
Length of the trajectory of the load suspension point	m	12.8401	12.8816	12.2126
Length of the load path	m	12.868	12.9048	12.2364
Movement duration	s	17.8862	25.7081	16.4959
RMS value of trolley acceleration	m/s²	0.00571288	0.00133661	0.202165
RMS value of boom acceleration	rad/s²	0.0170559	0.00939566	0.00534436
RMS value of load deflection in the boom plane	m	0.0204486	0.0148506	0.112766
RMS value of load deflection in the plane perpendicular to the boom plane	rad	0.00477456	0.00244031	0.00619055
RMS value of the driving force in the rope of the trolley movement mechanism	N	661.355	635.491	341.55
RMS value of the driving torque of the crane slewing mechanism	N·m	33073.6	20913.2	15159.5
RMS power value of the crane slewing mechanism	W	2165.93	1272.9	364.899

2) The initial and final conditions of the load movement are determined, which describe the resting state of the system at the beginning and end of the movement. These conditions are expressed in terms of the coordinates of the load, trolley and boom, as well as their velocities. On this basis, the boundary conditions are further derived exclusively for the positions of the load and their higher time derivatives.

3) The problem of trajectory optimisation is set, which involves minimising the duration of load movement, subject to the given constraints on the maximum velocity of the trolley and the boom rotation. The original problem is reduced to an unconstraint optimisation problem by introducing penalty functions. Its solution is obtained as a superposition of polynomial functions.

4) The analysis of the optimisation results showed a significant positive impact of regularisation (λ=0.1), including a significant reduction in the amplitudes of load oscillations, a decrease in driving forces and moments, a reduction in the number of cycles of pendulum oscillations of the load, and the elimination of the

need to change the direction of movement of the trolley and boom. On the other hand, the handling time has increased by approximately 43.7%.

5) Reducing the length of the flexible suspension from 5 m to 2.5 m reduced the movement time, reduced peak driving torques and forces, but led to a significant deterioration in dynamic performance and a significant increase in power consumption of the trolley movement mechanism.

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Синтез оптимальних траєкторій переміщення вантажу стріловим краном

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Анотація. У статті розглядається проблема синтезу оптимальних траєкторій переміщення вантажу баштовим краном з метою підвищення продуктивності та безпеки його роботи. Розроблено динамічну модель системи, що включає стрілу, візок і вантаж, підвішений на гнучкому канаті, з урахуванням маятникових коливань вантажу. Для визначення законів руху стріли та візка, які забезпечують задану траєкторію вантажу, розв'язано обернену задачу кінематики. Запропоновано оптимізаційний підхід, спрямований на мінімізацію тривалості транспортування вантажу за умов дотримання кінематичних і динамічних обмежень, зокрема максимальних швидкостей руху візка та обертання стріли. Для забезпечення плавності руху та зменшення динамічних навантажень застосовано методи регуляризації, які дозволяють знизити амплітуду коливань вантажу та усунути небажані зміни напрямку руху механізмів. Чисельне розв'язання задачі оптимізації виконано за допомогою модифікованого методу рою частинок (VCT-PSO), який продемонстрував ефективність у підборі оптимальних параметрів траєкторії. Проведено аналіз впливу регуляризації та довжини гнучкого підвісу на динамічні характеристики системи, зокрема амплітуду коливань, пікові зусилля та моменти, а також тривалість руху. Результати показують, що використання регуляризації значно знижує динамічні навантаження та підвищує стабільність руху, хоча й збільшує час транспортування. Скорочення довжини підвісу зменшує час руху, але може призводити до зростання амплітуди коливань. Запропоновано новий інструмент аналізу динаміки системи в координатах відхилень та різниці швидкостей, який забезпечує наочну оцінку коливального процесу та підтверджує виконання граничних умов. Отримані результати можуть бути використані для вдосконалення систем керування баштовими кранами та підвищення їхньої ефективності на будівельних майданчиках.

Ключові слова: баштовий кран, траєкторія вантажу, обернена кінематика, оптимізація, динаміка маятника, VCT-PSO, регуляризація.