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Theoretical research on the parameters of a vertical shaft rotary crusher

Yevhen Mishchuk[1], *Oleksandr Dyachenko*[2]

[1,2]Kyiv National University of Construction and Architecture,
Air Force Avenue, 31, Kyiv, Ukraine, 03037
[1]mischuk.ieo@knuba.edu.ua, https://orcid.org/0000-0002-1888-3687,
[2]diachenko.os@knuba.edu.ua, https://orcid.org/0000-0001-8199-2504

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Abstract. Stone is one of the most widespread and important natural resources, which are actively used in various fields of activity. Energy costs for the working processes of machines for the production of building materials can be significant. In addition, constant costs for maintenance, spare parts and power also impose an additional financial burden on companies producing building materials. Considering these factors, the issue of optimizing and improving such machines is extremely important. Increasing their efficiency and reducing energy costs will significantly reduce the cost of manufacturing building materials, as well as reduce the environmental burden.

A vertical shaft rotary crusher belongs to the class of centrifugal impact machines and works on the principle of material destruction by free impact in the field of action of centrifugal forces. In such crushers, several destruction mechanisms are implemented simultaneously. The interaction of these mechanisms ensures a very high crushing rate and obtaining a high-quality product. An important advantage of such crushers is that they work effectively at high rotor rotation speeds. The increase in the linear speed of the rotor flow enhances the impact effect, which makes it possible to destroy even very strong materials with relatively low energy consumption. At the same time, work on the principle of free impact ensures a uniform effect on each particle, which avoids excessive grinding and reduces the amount of dust fraction.

The paper considers the critical velocity of a material particle when it leaves the rotor. The equation of particle motion is analyzed. The influence of the physical and mechanical properties of the material on the critical velocity of particle exit is studied. Dependencies for determining the productivity and power of a rotary crusher are considered.

Keywords: vertical shaft rotary crusher, critical particle ejection velocity, productivity, power, relative particle ejection velocity, friction coefficient.

INTRODUCTION

A vertical shaft rotary crusher belongs to the class of centrifugal impact machines and operates on the principle of material destruction by free impact in the field of action of centrifugal forces [1]. Unlike traditional rotary impact crushers, where destruction occurs mainly on the working elements of the rotor, in vertical impact units the rotor itself performs almost exclusively the function of a flow accelerator, and the actual crushing zone is transferred to the periphery of the working chamber [2]. The rotor is fixed on a vertical shaft, which has special accelerating elements - blades, ribs or guide channels, which form the optimal trajectory of particle movement [3]. The material is fed strictly to the center of the rotor, after which the pieces are accelerated to high speeds and thrown out onto the reflecting surfaces where its destruction occurs [4].

In such crushers, three destruction mechanisms are implemented simultaneously: the impact of a piece on a solid obstacle, the mutual impact of particles with each other in a centrifugal flow, and contacts with moving elements of the rotor during acceleration [5]. The interaction of these mechanisms provides a very high crushing ratio and the production of high-quality cubic grains, which is especially appreciated in road construction and in the production of concrete with increased requirements for grain composition [6], [14].

The main design and technological difference of centrifugal impact crushers is the almost complete transfer of the impact load from the rotor to the stationary peripheral surface [7]. Due to this, the rotor does not require massive armor and heavy impact elements, and the wear of its parts is minimal - the main load falls on the reflecting system [8]. Depending on the surface chosen to receive the impact, two conceptual modes of operation are distinguished: “stone on metal” and “stone on stone” [9]. In the first case, the material hits steel armor plates, which ensures maximum grain opening and greater product controllability [10]. In the second, the fragments hit a layer of natural stone material that accumulates on the periphery of the chamber, creating a self-crushing cushion [11]. This mode provides minimal metal wear, perfect cuboid, and increased stability of work on abrasive rocks [12].

GOAL AND PROBLEM STATEMENT

Theoretical research on the parameters of a vertical shaft rotary crusher.

MAIN PART

The speed of particle escape from the rotor determined primarily by the peripheral speed of the tips of its blades [13]. However, at the same peripheral speed, the actual escape speed varies depending on the geometry of the rotor - the shape and location of the acceleration blades - and the coefficient of friction of the material on the acceleration surface. For a rotor with radial blades without taking into account friction, the theoretical escape speed (V_v) exceeds the peripheral speed of the tips of the blades $V_{k.sh.}$ by 1.41 times.

The actual velocity of particles leaving the rotor is always lower than the theoretically possible one due to frictional resistance, which slows down the movement of the material along the blade. The effect of friction taken into account by the coefficient m, the value of which varies within the range of 1.25–1.62.

Blade circular speed:

$$V_{k.sh.} = \frac{\pi n R}{30} , \tag{1}$$

where n – frequency rotor rotation, rpm; R – the radius of the circle described by a point located at the end of a rotor blade, m.

Thus, the particle exit velocity can be estimated from an empirical relationship that takes into account the effect of friction and the design parameters of the rotor:

$$V_v = \frac{\pi n R m}{30} , \tag{2}$$

The source [4] states that the critical impact velocity of a piece of crushed material can be determined based on the empirical dependence:

$$\upsilon_{kr} = K_\alpha \sqrt{\frac{2\sigma_p \Delta d}{0,98 \rho d_{kp}}} , \tag{3}$$

where σ_p- yield strength of rock, MPa; ρ - material density, kg/m^3; υ_{kr} - impact speed, which depends on the rotor speed, m/s; d_{kr} - critical diameter of crushed material, m; Δd – deformation of a material that is necessary for its destruction; K_α – coefficient that takes into account the type of impact of pieces of material on the impact ring (with a direct hit $K_\alpha = 1$, with an oblique blow $K_\alpha = 1.8$-2.2).

On the other hand, when accelerated by a bladed rotor, the particle participates in two movements: a) circular (translating) together with the bladed wheel; b) relative (relative to the blade).

The position of the straight blade determined by the relationship:

$$r_n / R = \sin \varphi = \cos \alpha , \tag{4}$$

where: r_n is radius normal to the blade; R is blade end point radius.

At some distance from the center of the blade at its initial section (S_o) material is deposited under the action of frictional force. The size of this area is determined by the ratio $S_0 = r_n f$, where f is coefficient of friction.

The equation of motion of a particle is based on the equality ma=ΣF$_i$

Then the equation of motion will have the following form [2]:

$$\frac{d^2l}{dt^2}+2\omega f_r\frac{dl}{dt}+\omega^2 l\times k_1=-\omega^2 r_n\times k_2-f_m g\ , \quad (5)$$

where: dl is elementary movement of a particle along a blade over time dt; ω is rotor angular velocity; f_r is coefficient of friction between particles; f_m is coefficient of friction of the particle on the rotor walls; k_1, k_2 is coefficients, which are defined as:

$$k_1=\left(\frac{(f_r\cos\varphi-\sin\varphi)\tan\varphi}{\sqrt{1+\tan^2\varphi}}\right); \quad (6)$$

$$k_2=(f_r\cos\varphi-\sin\varphi). \quad (7)$$

The equation of motion (5) describes the dependence of the absolute velocity of the particle at the exit on the geometry of the chamber, the rotor speed, and the coefficients of friction between individual particles and the particle and the rotor walls. When compiling the equation of motion, the relative velocity of the particle considered, that is, along the wall of the rotor chamber.

The relative velocity of a particle defined as [2]:

$$v_{rel}=-k_3k_4\omega\,|\,e^{-k_4\omega t}-e^{k_5\omega t}\,|, \quad (8)$$

where: k_3,k_4,k_5 some coefficients.

In work [3], the relative speed of movement is determined based on the path traveled by the particle, and the simplification is also adopted that the particle-to-particle and particle-to-metal friction coefficients are equal. In this case, the dependence for the relative velocity of the particle will be of the form:

$$v_{rel}=\frac{\omega(\sqrt{l^2-r_n^2}-r_n f)}{\sqrt{f^2+1}+f}, \quad (9)$$

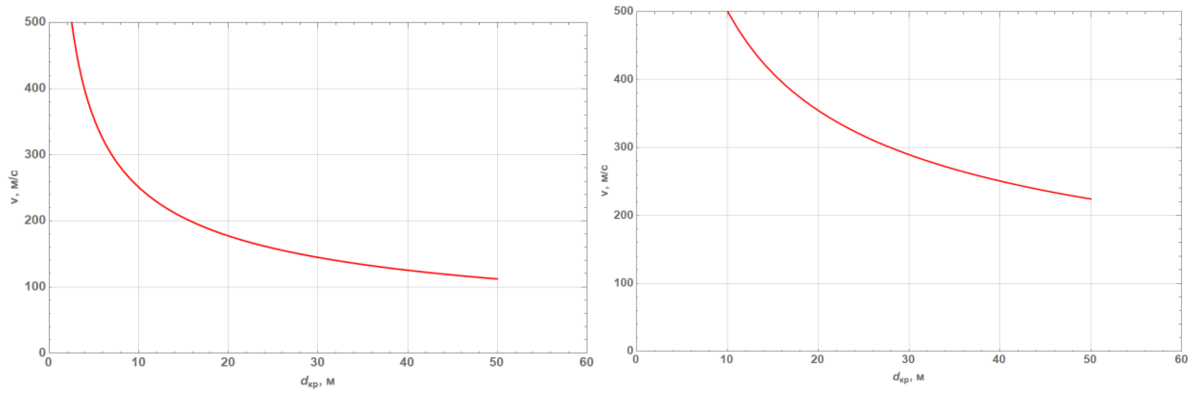
Let us investigate the influence of the size of the material and its strength on the required velocity of particle exit from the rotor based on the dependence (3).

In sources [4], [5] it is noted that the required minimum value of the parameter Δd starts from 3-5 mm. As a crushed material, we will first consider granite with a tensile strength of σ_r =200 MPa and specific density ρ= 2600 kg/m^3. Fig. 1 shows graphs of the critical particle velocity for direct and oblique impacts.

As can be seen from the graph, it can be noted that with an oblique impact, the speed required for the destruction of the particle is much higher. Let us write down the values of the critical speed of destruction with a direct impact for different sizes of the material: 1) 10 mm– 250 m/s; 2) 40 mm – 125 m/s.

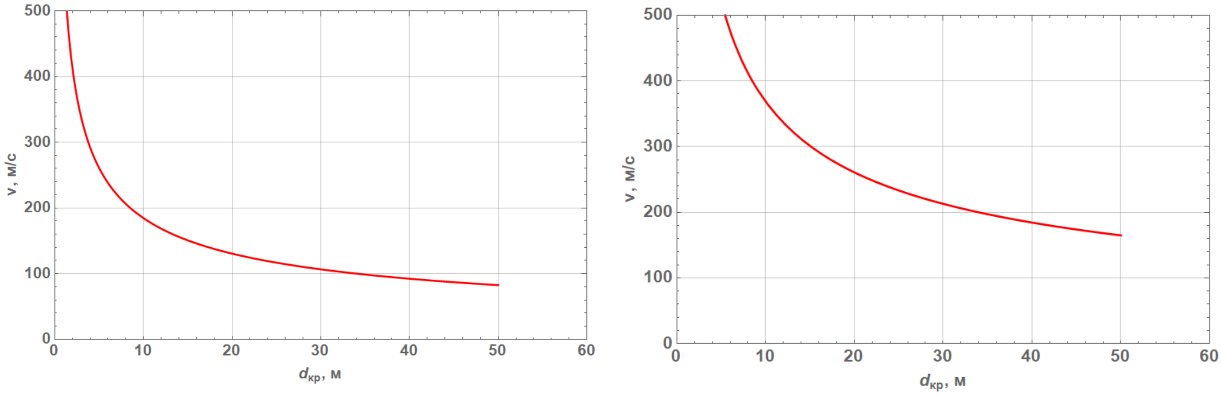
Next, we will consider medium-strength sandstone - σ_r =100 MPa and ρ = 2400 kg/m^3. The corresponding graphs are presented in Fig. 2.

When sandstone is destroyed, the critical velocity values for a direct impact are: 1) 10 mm – 184 m/s; 2) 40 mm – 92 m/s.



a *b*

Fig. 1. Graphs of the critical velocity of a granite particle: *a* – direct hit; *b* – oblique strike



a *b*

Fig. 2. Graphs of the critical velocity of a sandstone particle: *a* – direct hit; *b* – oblique strike

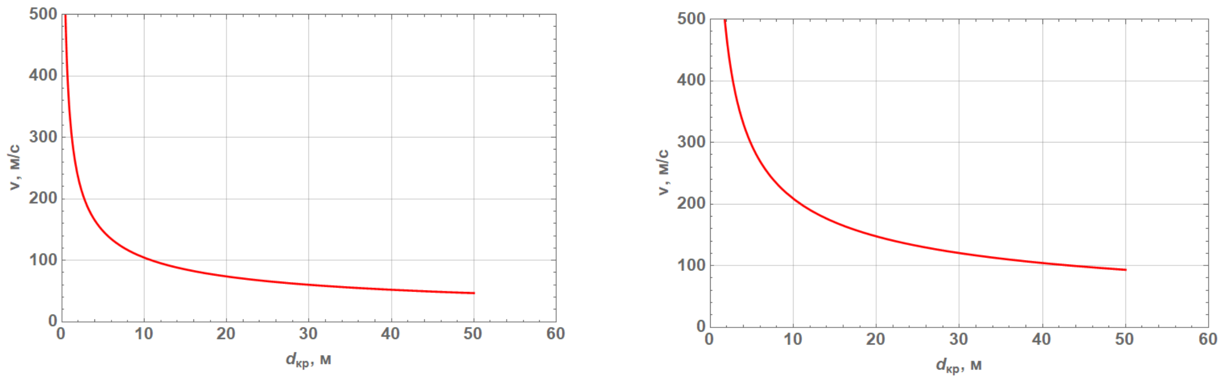
We build the same graphs for a material with a strength of 32 MPa, concrete of class C20/25 (B25, M350), ρ = 2400 kg/m^3 Fig. 3.

According to the destruction of concrete C20/25 following values of the critical velocity in a direct impact correspond: 1) 10 mm – 104 m/s; 2) 40 mm – 51 m/s.

Separately, we will plot graphs of the influence of material strength on the critical failure rate. It should be noted here that depending on (3.3) includes two parameters that determine the material of the fracture. This fact should be taken into account when constructing graphs, Fig. 4.

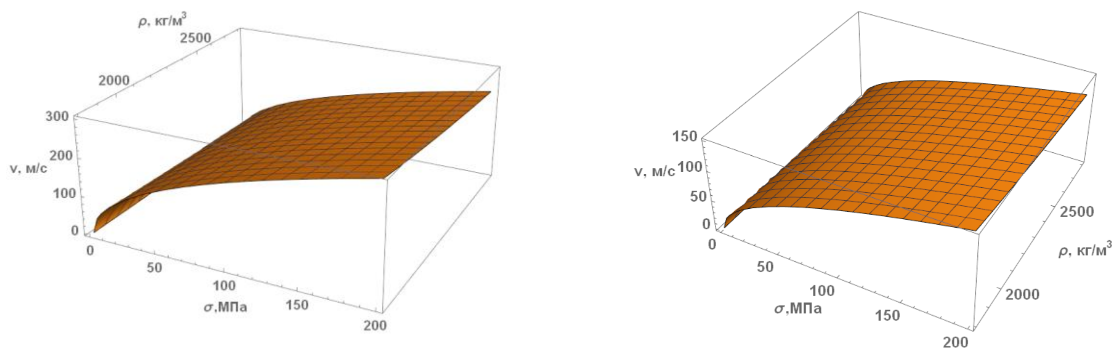
To determine the approximate size of the crusher outlet, we will use the known dependence:

$$S = 0{,}1D_r \qquad (8)$$



a *b*

Fig. 3. Critical velocity graphs for a concrete particle C20/25: *a* – direct hit; *b* – oblique strike



a *b*

Fig. 4. Graphs of the critical velocity of a material particle depending on the change in its physical and mechanical properties upon direct impact: *a* – particle size 10 mm; *b* – particle size 40 mm

where D_r is rotor diameter.

The calculation of the crusher's performance performed using the formula, m^3/hour:

$$Q = Q_0 \frac{L_p D_p^{1,5}}{\upsilon_p^{0,35} z^{0.5}} k_\beta k_D k_S k_r k_\sigma k_B \;, \tag{9}$$

where Q_0 = 165 m^3/hour – table crusher performance without taking into account the cascade (under cascade mode conditions Q_0 = 485 m^3/hour) and with the following accepted parameters – $D_r=L_r=v_r=z=1$; β=0; D->0;S=0; σ_r ->0. In turn, the coefficient k_β takes into account the influence of the angle of installation of the reflector plate and is determined based on the following empirical dependence:

$$k_\beta = 1 - 0,49\sin\beta + 4,7\sin^2\beta, \tag{10}$$

where: $\beta=20^0$ – installation angle of the first reflector plate; k_D – the coefficient that adjusts the crushing performance depending on the size of the raw material pieces is calculated using the empirical formula:

$$\sigma = \frac{D_{av}}{D_r} \Rightarrow \kappa_D = 0,3; \tag{11}$$

where; D_c – average size of the crushed product; k_S – the size of the rounding of the front edge of the hammer is taken into account in the calculations using coefficient, the value of which is obtained from empirical relationships:

$$\kappa_S = 1 + 1,9\varepsilon \;, \tag{12}$$

At the same time

$$\varepsilon = \frac{S}{D_r}, \tag{13}$$

where: k_r=0,85 – coefficient that takes into account the effect of the rounding of the leading edge of the bat; k_σ – the coefficient, which takes into account the influence of the physical properties of the crushed material on the crushing process, is determined by the dependence:

$$k_\sigma = 1 - \frac{C_\sigma}{700}, \tag{14}$$

where C_σ is strength criterion

$$C_\sigma = \frac{\sigma_r}{\gamma_0 D_r}, \tag{15}$$

where: k_B=0,86 – the coefficient that corrects the crushing process depending on the wavy shape of the outer surface of the hammer is determined empirically.

It should be noted here that dependence (5) gives an approximate value of productivity, which is associated with a large set of empirical parameters, as well as with not taking into account the design features of a rotary crusher with a vertical shaft.

The power consumed by the centrifugal crusher is used to overcome friction in bearings and transmission mechanisms, as well as to directly carry out the material crushing process.. That is:

$$N = N_x + N_{spec} \cdot Q, \tag{16}$$

where N – total power consumed by the crusher, kW; N_x – power consumed during idle operation of the crusher, $N_{spec.}$ – crusher power used directly for the crushing process at a given performance 1 t/hour; Q – crusher performance in open cycle conditions, t/hour.

The power consumed at idle is determined from the following dependence:

$$N_x = N_1 \left(\frac{n}{1000} \right)^b, \tag{17}$$

where: N_1 is power at rotor speed of 1000 rpm; *b* – exponent, the value of which varies from 1,1 to 1,8 depending on the rotor design.

The specific energy consumption depending on the particle ejection from the rotor is determined from the following dependence:

$$E_{spec} = \frac{\left(m_{f=0} V_{k.sh.} \right)^q}{7200}, \tag{18}$$

where $V_{k.sh.}$ – rotor rotational speed, m/sec; q – exponent, varies from 2 to 2,15.

In the source [2] it is proposed to determine the power of the electric motor of a rotary crusher by the formula, kW:

$$N=\frac{W_{cr}Q(i\text{-}1)}{D_{av}\eta_{cr}\eta_{p}}, \qquad (13)$$

where: W_{cr}= $18 \cdot 10^{-3}$ kWh/m^3 is specific energy index [1]; Q is productivity, m^3/hour; i is degree of crushing; η_{cr} is machine efficiency, η_{cr}=0,85; η_{p}=0,95 is drive coefficient.

CONCLUSIONS

Based on the research, it can be note that the calculation of the parameters of the mechanical mode of rotary crushers is based on empirical equations. When considering the speed of particle exit from the rotor, a significant influence of the rotor geometry and friction coefficients was established. Under the condition of the departure of material particles from the rotor, which have low strength and a size of 40 mm and above, the change in speed does not significantly affect their destruction and is more linear in this size range. For stronger materials, the critical fracture velocity is nonlinear and increases significantly with decreasing material size. Direct impact of particles colliding with the crushing chamber lining is more advantageous in terms of energy efficiency of the fracture process compared to oblique impact.

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Теоретичне дослідження параметрів вертикальної роторної дробарки

Євген Міщук[1], *Олександр Дьченко*[2]

[1,2] *Київський національний університет будівництва та архітектури*

Анотація. Камінь є одним із найпоширеніших та найважливіших природних ресурсів, який активно використовується в різних сферах

діяльності. Витрати енергії на робочі процеси машин для виробництва будівельних матеріалів можуть бути значними. Крім того, постійні витрати на технічне обслуговування, запасні частини та електроенергію також накладають додаткове фінансове навантаження на компанії, що виробляють будівельні матеріали. Враховуючи ці фактори, питання оптимізації та вдосконалення таких машин є надзвичайно важливим. Підвищення їхньої ефективності та зниження витрат енергії значно знизить собівартість виробництва будівельних матеріалів, а також зменшить навантаження на навколишнє середовище.

Вертикальна роторна дробарка належить до класу відцентрових ударних машин і працює за принципом руйнування матеріалу шляхом вільного удару в полі дії відцентрових сил. У таких дробарках одночасно реалізовано кілька механізмів руйнування. Взаємодія цих механізмів забезпечує дуже високу швидкість дроблення та отримання високоякісного продукту. Важливою перевагою таких дробарок є те, що вони ефективно працюють при високих швидкостях обертання ротора. Збільшення лінійної швидкості потоку ротора посилює ударний ефект, що дозволяє руйнувати навіть дуже міцні матеріали з відносно низькими енерговитратами. Водночас робота за принципом вільного удару забезпечує рівномірний вплив на кожну частинку, що дозволяє уникнути надмірного подрібнення та зменшує кількість пилової фракції.

У статті розглядається критична швидкість частинки матеріалу при її виході з ротора. Проаналізовано рівняння руху частинки. Досліджено вплив фізико-механічних властивостей матеріалу на критичну швидкість виходу частинок. Розглянуто залежності для визначення продуктивності та потужності роторної дробарки.

Ключові слова: вертикальна роторна дробарка, критична швидкість викиду частинок, продуктивність, потужність, відносна швидкість викиду частинок, коефіцієнт тертя.